

# Qualitative Reasoning beyond the Physics Domain: The Density Dependence Theory of Organizational Ecology\*

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**Abstract:** Qualitative reasoning is traditionally associated with the domain of physics, although the domain of application is, in fact, much broader. This paper investigates the application of qualitative reasoning beyond the domain of physics. It presents a case study of application in the social sciences: the density dependence theory of organizational ecology. It discusses how the different nature of soft science domains will complicate the process of model building. Furthermore, it shows that the “model building” process can also help making theoretically important decisions, and, as a result, have an impact on the original theory. This will require a shift in focus from the “model simulation” process towards the “model building” process.

## 1 Introduction

During the last decades, Qualitative Reasoning (QR) has been an active area of research. The field has reached a consensus on the main issues. As a consequence of this, the time has come to think about extending its domain of application. QR is traditionally associated with the physics domain. This domain of application has been so dominant that qualitative reasoning is often called qualitative physics, for example Forbus (1988). Extending the domain of QR prompts an interesting question: is the dominant relation between QR and physics based on ontological arguments?

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The application of QR outside the traditional physics domain seems, indeed, possible. Kuipers (1994)<sup>1</sup> lists applications in biology (irreversible population change, predator-prey ecology), chemistry (chemical engineering), economics (supply and demand, micro-economics), and medicine (glaucoma, drug metabolism). The answer to our question appears to be negative, there are no ontological reasons to explain why the majority of QR-research deals with physics applications. Does the lack of ontological arguments mean that the relation between QR and physics is purely accidental? Probably not, there may be other, pragmatical arguments to explain this relation. A plausible explanation for the historical choice to reason about physical systems is the formal, well-understood nature of the physics domain. This indicates that QR outside physics, albeit possible, may still not be the very same as QR inside physics. The different nature of the domain may require different emphases. This prompts another question: does the application of QR outside physics require a change in methodology?

It is with these questions in mind that we performed the case study reported in this paper. The intention of the paper is to investigate the differences between physics and other domains in the context of QR. Therefore, a natural choice of domain for this case study is a “soft” science domain. The soft sciences are in many respects the opposite of physics, in being highly non-formal, less well-understood. We have chosen to build a QR-application in the social sciences.

This paper is organized in the following way. Section 2 gives a short introduction to the framework for qualitative reasoning that we used for

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<sup>1</sup>Several other authors have also reported applications of QR outside physics.

our research. Section 3 and 4 introduce the domain of our case study, i.e., the density dependence theory of organizational ecology. Section 5 describes the qualitative density dependence model, and summarizes its qualitative behavior prediction. Section 6 lists the results that were obtained during the construction and use of the qualitative model. And finally, in section 7 we evaluate our case study in retrospect, trying to answer the questions posed above.

## 2 Representational Context

The model of this paper is implemented in a domain-independent qualitative reasoning shell called GARP (Bredeweg, 1992). GARP incorporates many features of the component-centered (de Kleer and Brown, 1984), and the process-centered (Forbus, 1984) approaches in QR. Initial conditions are described in *case models*, the theory itself in *model fragments* (consisting of *conditions* and *givens*). Case models and model fragments can be expressed in terms of: *entities* (like liquid), *quantities* (like amount), *values* and *derivatives* (like  $\{-, 0, +\}$ ), and *dependencies* (like (in)equalities, proportionalities, influences, etc.); or in terms of other model fragments.

The behavior of a system during a particular time period is described by the set of applicable model fragments. The behavior over time periods is determined by the application of transition rules between states of behavior.

## 3 The Density Dependence Theory

Mainstream organizational theories regard organizations as agents that adapt rationally to changing environments (Thompson, 1967; Mintzberg, 1979). These theories describe organizations from an *individual* viewpoint. Complementarily, a change in environmental resource conditions affects the whole *population* of organizations. For example, if resource conditions deteriorate, the total population of organizations will decline (despite the efforts of individuals to avoid this fate). Organizational ecology (Hannan and Freeman, 1989) describes the process by which organizational populations grow and decline due to changing environmental conditions. Organizational ecology abstracts from the rational behavior of individuals, populations are solely depen-

dent on the environmental conditions.

The density dependence theory (Hannan and Carroll, 1992) is at the heart of organizational ecology: it describes the dynamics that underlie the growth of an isolated population as a function of the population's density. It serves as a base model for other parts of organizational ecology that investigate the demographic behavior of different (sub)populations under changing environmental conditions (e.g., niche strategists, life history strategists) or during reorganization (e.g., the inertia-fragment (Hannan and Freeman, 1989; Péli et al., 1994)). The density dependence theory assumes that the founding and mortality rates of a population are affected by two opposing forces: by the degree of legitimation that the population enjoys and by the intensity of competition between the members of the population.

*Legitimation* reflects the institutional standing of the population. A high level of legitimation means that the organizational population has the status of a taken-for-granted solution to given problems. Organizations of high legitimation are desirable partners for other organizations when making exchange relations. Moreover, founding new organizations in a highly legitimated population is also easier. The theory assumes that the founding rate is directly proportional to the legitimation of the population, and that the mortality rate is inversely proportional to it. Legitimation increases monotonically with density. The beneficial effect of growing density is especially important when there are only a few organizations in the environment. If organization density is high already, then the founding of an additional organization does not improve the population's institutional standing significantly. The higher the density of a population is, the smaller is the legitimating effect of additional organizations.

An intensifying *competition* between the member organizations of the population increases the mortality. It also decreases the founding rate of the population: managers are reluctant to initiate new organizations if the chance of success is low. The theory assumes that the intensity of competition is directly proportional to the mortality rate and inversely proportional to the founding rate. Since competition is about resources, the intensity of competition increases with density. The density dependence theory claims that increasing

density intensifies competition at an increasing rate.

The beneficial effects of legitimation prevail at low densities, while the negative effects of competition dominate if density is high. As a result, the demographic rates change with density in a *non-monotonic* way. The founding rate increases over the lower density range and decreases above a certain value. On the other hand, the mortality rate decreases at low densities, and increases later. When the two rates become equal, the population reaches its equilibrium size: this value is called the *carrying capacity* of the given resource environment.

Hannan and Carroll (1992, Chapter 2) give the following description of the intuitive theory specified above:

**Competition** The intensity of competition,  $C$ , increases with density,  $N$ , at an increasing rate. That is,  $C = \varphi(N)$ ; and  $\varphi' > 0$  and  $\varphi'' > 0$ .

**Legitimation** The intensity of legitimation,  $L$ , increases with density at a decreasing rate. That is,  $L = v(N)$ ; and  $v' > 0$  and  $v'' < 0$ .

**Founding Rate** The founding rate of an organizational population,  $\lambda$ , is inversely proportional to the intensity of competition within the population, and directly proportional to the legitimation. That is,  $\lambda \propto 1/C$  and  $\lambda \propto L$ .

**Mortality Rate** The mortality rate of an organizational population,  $\mu$ , is directly proportional to the intensity of competition within the population, and inversely proportional to the legitimation. That is,  $\mu \propto C$  and  $\mu \propto 1/L$ .

For environments with a positive carrying capacity, it is assumed that legitimation exceeds competition at low densities, that is,  $L_i \geq C_i$  for  $i < N$ . To avoid negative founding and mortality rates, the range of the legitimation and competition functions also has to be confined to non-negative numbers. Since legitimation and competition occur in the denominator of the mortality and the founding rate, respectively, their value cannot be zero either. The theory assumes that  $L > 0$  and  $C > 0$ .

The legitimation and competition functions are depicted in Figure 1. Competition increases with

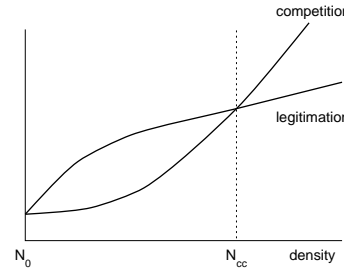


Figure 1: *Legitimation and competition as functions of density.*

density at an increasing rate, and legitimation increases with density at a decreasing rate. In this figure,  $N_0$  denotes the initial point, and  $N_{cc}$  denotes the point where legitimation and competition are equal.

## 4 Applying The Theory

Our aim is to simulate the growth pattern of populations, in other words, how the density of a population changes over time. Hannan and Freeman (1989) use the Lotka-Volterra definition of growth rate,  $\rho$ , as the difference between founding and mortality rates of the population. That is,  $\rho = \lambda - \mu$ . The growth rate can be calculated from legitimation and competition directly: if  $\lambda = L/C$  and  $\mu = C/L$  then  $\rho = L/C - C/L$ .<sup>2</sup>

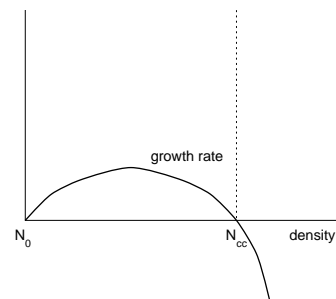


Figure 2: *Growth rate as a function of density.*

Figure 2 depicts the growth rate of an organizational population, based on the legitimation and competition functions in Figure 1.

<sup>2</sup>In fact, founding and mortality rates may differ by a factor. That is,  $\lambda = a * L/C$  and  $\mu = b * C/L$ . We ignore these factors to allow for the direct calculation of the growth rates, because the use of a simple model helps to convey the main point of our argument. Moreover, if the factors  $a$  and  $b$  are given we can rescale the  $C$  and  $L$  functions to  $C^*$  and  $L^*$ , so that  $\lambda = L^*/C^*$  and  $\mu = C^*/L^*$ . The rescaled  $C^*$  ( $L^*$ ) still satisfies the criteria of increasing at an increasing (decreasing) rate.

We can now investigate the theory’s predictions about the change of population size. If the en-

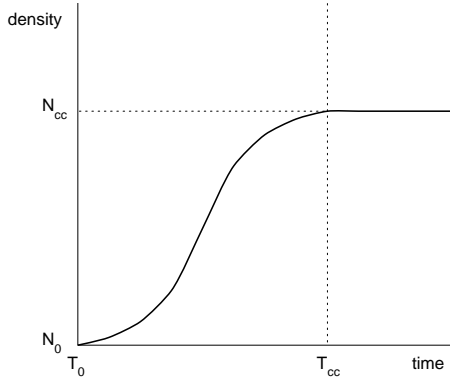


Figure 3: *Expected behavior for an empty environment.*

vironment is initially empty (Figure 3), then the density dependence theory predicts a sigmoid (or S-shaped) population growth. This growth pattern is similar to the well-known *logistic curve* of the Lotka-Volterra model.

## 5 Qualitative Density Dependence

In section 4 we have established that growth rate  $\rho = L/C - C/L$ . Having only positive legitimation and competition values, this means that  $\rho > 0$  if  $L > C$ ,  $\rho = 0$  if  $L = C$ , and  $\rho < 0$  if  $C > L$ . The sign of  $L - C$  equals the sign of  $\rho$ . The qualitative model of growth rate therefore is  $\rho = {}_Q L - C$ .<sup>3</sup>

Section 3 also gives constraints on legitimation and competition. Legitimation is increasing with density at a decreasing rate,  $\delta L/\delta N > 0$  and  $\delta^2 L/\delta N^2 < 0$ . Competition is increasing with density at an increasing rate,  $\delta C/\delta N > 0$  and  $\delta^2 C/\delta N^2 > 0$ . But instead of derivatives to density, derivatives to time are needed. If legitimation is greater than competition (a growing population) density will increase with time. This means that we can use the derivatives listed above. This qualitative behavior can be modeled as follows:  $L \propto_Q N$ ,  $\delta L \propto_Q 1/N$ ,  $C \propto_Q N$ , and  $\delta C \propto_Q N$ .

If legitimation is smaller than competition (a declining population) density will decrease with time. This means that we have to read the top

<sup>3</sup>To denote the difference between qualitative dependencies and their mathematical counterparts, we use “ $=_Q$ ” instead of “ $=$ ” for equalities, “ $\propto_Q$ ” instead of “ $\propto$ ” for proportionalities, and “ $I$ ” for influences.

half of Figure 2 from the right to the left. In this part, competition is decreasing with decreasing density at an *increasing* rate, and legitimation is decreasing with decreasing density at a *decreasing* rate. This means that the second-order derivatives of  $L$  and  $C$  change. This qualitative behavior can be modeled as follows:  $L \propto_Q N$ ,  $\delta L \propto_Q N$ ,  $C \propto_Q N$ , and  $\delta C \propto_Q 1/N$ .

If density is increasing, it has a positive effect on the second order derivative of competition and a negative effect on the second order derivative of legitimation. If density is decreasing these effects are reversed. In the equilibrium points (legitimation equals competition, i.e.,  $N_0$  and  $N_{cc}$ ), density is not changing, making this difference disappear.

$$N \quad I^+ \quad \rho \quad (1)$$

$$L \quad I^+ \quad \delta L \quad (2)$$

$$C \quad I^+ \quad \delta C \quad (3)$$

$$L \quad \propto_{Q+} \quad N \quad (4)$$

$$\delta L \quad \propto_{Q-} \quad N \quad (5)$$

$$(\delta L \quad \propto_{Q+} \quad N) \quad (6)$$

$$C \quad \propto_{Q+} \quad N \quad (7)$$

$$\delta C \quad \propto_{Q+} \quad N \quad (8)$$

$$(\delta C \quad \propto_{Q-} \quad N) \quad (9)$$

$$\rho \quad =_Q \quad L - C \quad (10)$$

Figure 4: *Dependencies of qualitative density dependence.*

Figure 4 summarizes the model. Dependencies 1 to 3 are included for technical reasons, they allow the use of higher-order derivatives by modeling them as normal values. Dependencies 4 and 5 ensure that legitimation is increasing with density at a decreasing rate (if legitimation is smaller than competition, dependency 6 replaces 5). Dependencies 7 and 8 ensure that competition is increasing with density at an increasing rate (if legitimation is smaller than competition, dependency 9 replaces 8). Dependency 10 calculates the growth rate from the legitimation and competition values.

We have now modeled the causal chain of the density dependence theory: i) the trade-off between competition and legitimation causes a certain growth rate, ii) the growth rate will affect

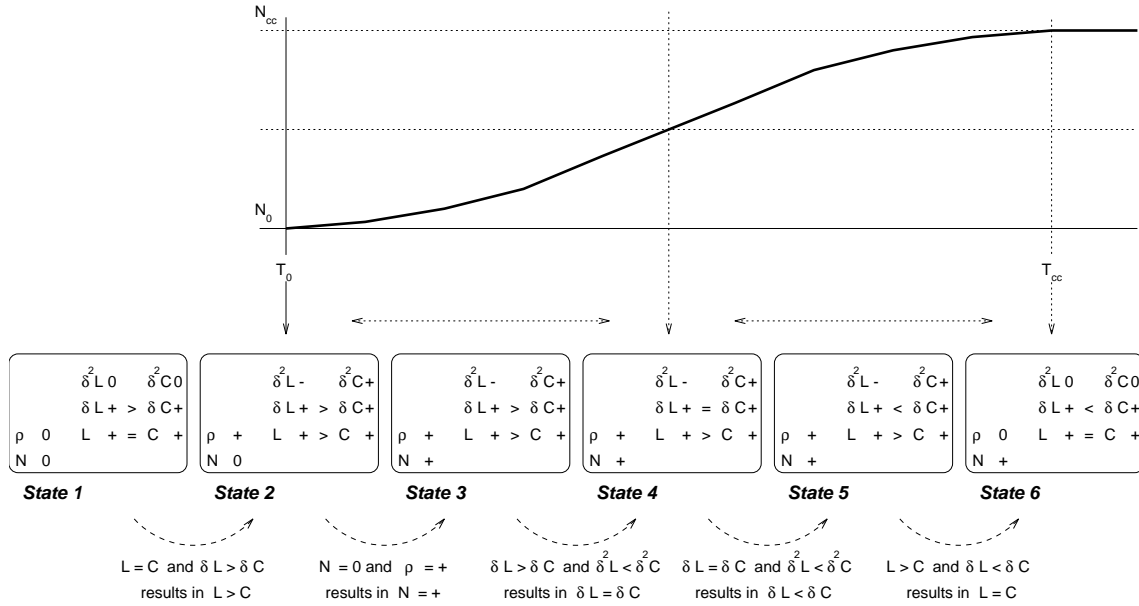


Figure 5: *Case A: An empty environment.*

the density of the population, iii) the change in density, in its turn, will affect competition and legitimation, iv) etcetera.

Using this qualitative density dependence model, the qualitative reasoning shell GARP can make behavior predictions of the following case models:

**Case A: An empty environment** (see Figure 5). In this scenario,  $N = 0$ ,  $L = C$ , and  $\delta L > \delta C$ . State 1 is (just before) the point  $T_0$ , state 2 corresponds to  $T_0$ , state 3 to the interval between  $T_0$  and  $T_1$ , state 4 to the point  $T_1$ , state 5 to the interval between  $T_1$  and  $T_2$ , and finally, state 6 to point  $T_2$ .

**Case B: A population in equilibrium** (see Figure 6). In this scenario,  $N > 0$ ,  $L = C$ ,  $\delta L = 0$ , and  $\delta C = 0$ . Case B results in a steady state. The density of the population is at its carrying capacity.

**Case C: An overcrowded environment** (see Figure 7). In this scenario,  $N > 0$ ,  $L < C$ , and  $\delta L > \delta C$ . State 1 corresponds to the interval before  $T_3$ , and state 2 to the point  $T_3$ .

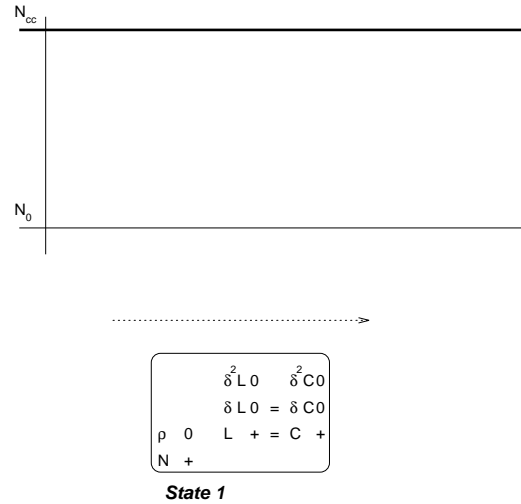


Figure 6: *Case B: A population in equilibrium.*

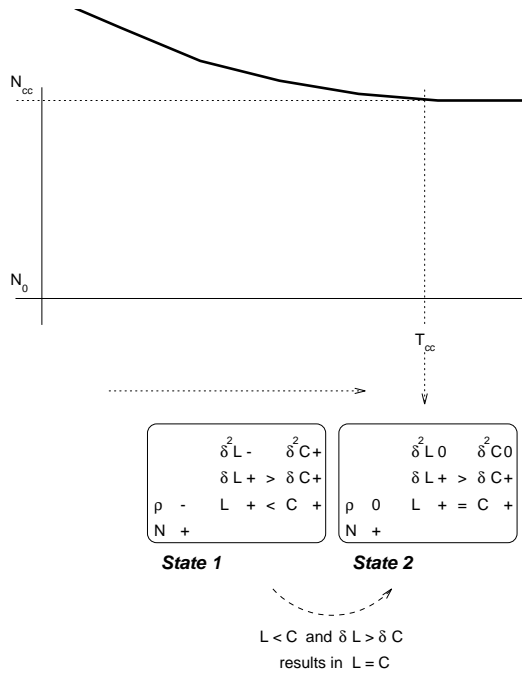


Figure 7: Case C: An overcrowded environment.

## 6 Results

Hannan and Carroll (1992) give a formal mathematical model of density dependence, as well as an explicit qualitative description using the proportionalities and the signs of the derivatives as summarized in section 3. The qualitative description provides the intuitions underlying the mathematical description of the theory. For building the qualitative model, we only used the qualitative description of the theory. This resulted in a intuitive model. It describes the core elements of the theory and abstracts from unnecessary detail. The model can be used learn the theory and the theory's predictions. The resulting qualitative density dependence model is able to derive the behavior predicted by the theory. The qualitative model is more general than the parametrical model (and the resulting quantitative simulations). A quantitative, parametrical model makes, out of necessity, various non-trivial assumptions about parameters, functions, etcetera.

During the modeling process we had to make several decisions of theoretical importance. These decisions reveal implicit assumptions underlying the theory. Making these assumptions explicit is an important contribution to the original theory. Apart from identifying hidden assumptions underlying the theory, the qualitative simulator was

also able to predict unidentified consequences of the theory. We will discuss some of these implicit assumptions and consequences in detail.

First, the theory gives no information about the derivatives of legitimation and competition at zero density. Therefore, it is possible that the derivative of competition is initially higher than the derivative of legitimation, i.e.,  $\delta C_0 > \delta L_0$  (as depicted in Figure 8). That is, the deriva-

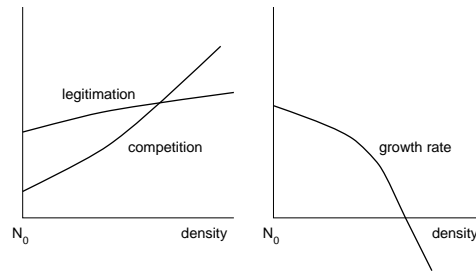


Figure 8: Monotonically growing population.

tive of growth rate is initially negative, resulting in a *monotonic* population growth. This is in conflict with the theory's claim that the demographic rates are *non-monotonic*. A constraint is needed on the initial derivatives of competition and legitimation: at zero density, the derivative of legitimation is greater than the derivative of competition ( $\delta L_0 > \delta C_0$ ).

Second, in state 3 of case A the simulator predicts a state transition from unequal derivatives of legitimation and competition to equal derivatives. Although this transition is likely to occur, it is not guaranteed to take place. In state 3,  $L > C$ ,  $\delta L > \delta C$ ,  $\delta^2 L < 0$ , and  $\delta^2 C > 0$ . There is no guarantee that  $\delta L$  will become equal to  $\delta C$ . If legitimation and competition behave as in Figure 9, the population will stay in state 3, that is,

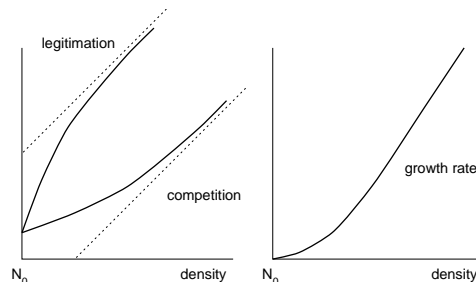


Figure 9: Forever expanding population.

it grows exponentially into infinity. This is clearly unintended: real-world's resources are always fi-

nite. This case can be avoided by assuming that competition exceeds legitimation after a certain density value, that is,  $C_i > L_i$  for  $i > N$  (recall that legitimation exceeds competition at low densities).<sup>4</sup>

Third, the decline of overcrowded populations is also captured by the density dependence theory.<sup>5</sup> If there are more organizations around than the carrying capacity of the environment, for instance due to migration, the density falls until the population reaches the carrying capacity (see Figure 10). The theory claims that the density

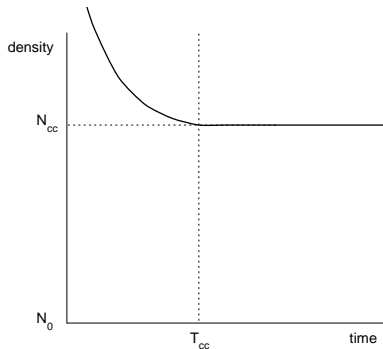


Figure 10: *Expected behavior for an overcrowded environment.*

dependence of organizational populations is non-monotonic. This is, indeed, the case for increasing populations. The decrease of populations manifests, in contrast, a monotonic pattern.

## 7 Discussion

As discussed in the previous sections, the density dependence theory of organizational ecology was successfully modeled as a QR-application. In accordance with existing research, our case study did not reveal any ontological arguments that would prevent the application of QR outside physics.

A full-fetched discussion of the metaphysics of QR is outside the scope of this paper. This discussion should focus on the level of *abstraction*. Physics has reached a high level of abstraction using terms such as *energy* or *gravity* as abstractions of underlying forces. These terms have an advanced nature (anyone who tried to explain to

<sup>4</sup>A different remedy, namely to impose an upper-bound on legitimation, is suggested in Péli (1993).

<sup>5</sup>This additional case model was found by hand, although it could have been discovered using a *total-environment* of states and transitions.

a child what *gravity* is will have noticed this). The non-physics domain of this paper and those of Kuipers (1994) have also reached a level of abstraction that is sufficient for building QR-applications.

Although feasible, the application of QR in a soft science domain is somewhat different from applying QR in physics. This difference seems to emerge from the different natures of both domains. There is one important difference between the domain of physics and a domain in the soft sciences. Not surprisingly, soft science domains are less well-understood, less formalized. In short, soft sciences lack the deep understanding of domain knowledge that characterizes physics. This is a subtle difference, but it has important methodological consequences.<sup>6</sup>

### 7.1 Finding the right model for the job

A first consequence is that building models on less well-understood, less formal domains will require more effort, and therefore will be more time consuming. Although the value of the “model simulation” process (Forbus, 1988) remains, the “model building” process must obtain a more prominent status.

There is another, deeper, consequence. In physics, terms such as *liquid flow* have a clearly established meaning (Hayes, 1985). In the soft sciences, the nomenclature is less developed; there is hardly any consensus. Terms like *legitimation* vary in their precise meaning among different theories. For example, the term legitimation denotes, in the density dependence theory, the taken-for-grantedness of an organizational population. On many occasions, the precise nature of a variable is only defined by the behavior it manifests in the context of a theory, and not by its label. As a consequence, the constructed domain models will not be very generic, they will only be reusable to a very limited extent.

In the traditional case (Falkenhainer and For-

<sup>6</sup>In the following, we will exaggerate this difference in understanding of the domains in order to make our arguments more clear. In reality, insufficient knowledge about the domain can play a role in all domains, including physics (as one reviewer put it: “Modeling in the physical sciences and in engineering is still very much an art”). Thus, insights in the way to handle insufficient domain knowledge are useful for QR in general.

bus, 1991), the use of a particular term, say *liquid flow*, induces the use of a particular model fragment specifying the behavior related to this term. But when terminology has no exact meaning this process is reversed: a needed model fragment determines the use of a certain term. The prediction of certain desired behavior, e.g., a sigmoid density function in the density dependence theory, requires the opposing force of two underlying causes. After constructing the model fragments with unlabeled variables, we start to look for sociological meaningful names, for example the terms legitimation and competition, for the two underlying causes.

Summarizing, the application of QR in soft science domains will require more effort in the model building process. Moreover, the constructed model fragments will only be reusable to a limited extent. Thus, it is less likely that the modeling process can be facilitated by existing libraries of generic model fragments.

## 7.2 Finding the right job for the model

The same fact that causes the model building process to be more troublesome, also makes it more worthwhile. Let us analyze what is disturbing the modeling process. Is it the “incompetence” of the model builder? Although the bounded rationality of a human model builder is certainly an important factor that tampers with the modeling process, there is no reason why the same model builder should be less competent when modeling a soft science theory. Is it the “incompetence” of the theory? We think so: during the modeling process many decisions have to be made, that have an impact on the original theory.

The theory can be ambiguous and allow for various interpretations. If these ambiguities are solved in the qualitative model, this solution corresponds also to an improvement of the original theory (see section 6 for examples).<sup>7</sup> The explication of the underlying structure of a theory will provide new theoretical insights. The model can reveal underlying assumptions, and thereby shed light on the theory’s domain of application. Furthermore, the simulator can identify unforeseen (and even counterintuitive) consequences of a theory, and thereby clarify the theory’s predic-

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<sup>7</sup>Note that handling incomplete knowledge is one of the strong points of QR.

tive and explanatory power. In short, the original theory will evolve in parallel with its qualitative model during the modeling process.<sup>8</sup>

Suppose we attempt to build the qualitative model of a premature, possibly imperfect theory. Instead of a straightforward translation, the model building will require decisions of theoretical importance. These decisions can be facilitated by simulation runs: various alternatives can be implemented and evaluated for their impact on the behavior prediction. The predicted behavior may not be in accordance with intuitions, logic, or empirical knowledge. These discrepancies will guide the model builder in the revision of the qualitative model (or of the expectations).<sup>9</sup> Moreover, these revisions will also apply to the original theory. In this way, the tedious “debugging” steps in traditional QR acquire a new character. They become experiments at the frontier of a science: every successful and every unsuccessful revision of the model may extend our knowledge about the theory.

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<sup>8</sup>Contemporary philosophy of science argues that theory development follows a cyclic pattern (Kuhn, 1962; Lakatos, 1976; Balzer et al., 1987). After the initial formation of a theory, it is repeatedly revised to account for anomalous observations (and may, in the end, be abandoned if the revision would require too radical changes, resulting in a paradigm shift). QR can play an important role in this diachronic structure of a theory, it allows us to recreate the theory-evolution process at a miniature scale.

<sup>9</sup>Several machine learning tools to support the evolutionary model building process (Falkenhainer and Rajamoney, 1988), and the diagnose/repair step (Bredeweg and Schut, 1993) have been reported.



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