Partial Deductive Closure: Logical Simulation and Management Science

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This research is part of a larger effort to build machine-based tools for developing scientific theories. In analogy with the *research process* in empirical research, we describe a *logical cycle* of theory development: (1) starting with an informal version of a theory, (2) then moving to its formal representation, (3) applying formal logic to investigate this representation, and (4) using the results as feedback for the update/revision of the original theory. A central aspect of the logical cycle is the detection of the (hidden) implications of a theory (called "partial deductive closure"). In this paper, we present an algorithm that performs the partial deductive closure for a relevant class of theorems, while filtering out trivial results. The algorithm is applied to an important organization theory, Organizational Ecology, and is shown to generate new theorems of interest.
(*Organizational Ecology; Theory-Building; Logic Modeling; Automated Deduction; Artificial Intelligence*)

1. Introduction

Computer simulation has become a standard tool in Management Science. A domain is represented in a formal language, and the properties of the domain are investigated through the properties of the formal representation (the "simulation model").

Traditionally, such representations used equational mathematics, e.g., numerical difference equations, as the formal language (Forrester 1961, Cohen et al. 1972, Burton and Obel 1984). Recent years, however, have seen various efforts to use "qualitative" or "declarative" representation languages instead (Baligh et al. 1990, Glorie et al. 1990, Blanning 1992, Carley and Prietula 1994, Péli et al. 1994, Kamps and Péli 1995). These efforts are usually inspired by progress in Artificial Intelligence—and by domains so complex that they defy numerical representation. The researcher may not know the numerical value of a variable, but still want to incorporate this variable in the model. For this reason, qualitative languages are an attractive alternative for numerical languages.

With the advent of expert systems, a large variety of qualitative languages became available. The common denominator of these languages is usually (a fragment of) First Order Logic (FOL), the best known formal logic. This commonality has focussed attention on the use of formal logic as a representation language, both in Management Science and elsewhere (Kimbrough and Lee 1988a and 1988b, Kimbrough 1990, Masuch 1992, Bhargava and Kimbrough 1994). As a *logic*, FOL has considerable expressive power and has useful features for developing better theories. For example, FOL provides precise criteria for theoretical *consistency* (is a theory contradiction-free?), *soundness* (are the explanations of a theory logically correct?) and *contingency* (is the theory falsifiable?).

By testing a theory for these properties, the researcher can develop better theories with the help of logic. The researcher formulates his theory in a logical language, and the computer investigates the logical properties of this representation. The logical representation itself could then play the role of a simulation model, where the computation of the "outcome" is done through logical inferencing.

Now, if one wants to spell out the logical consequences of a set of assumptions, then one is, technically

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speaking, working on the "deductive closure" of this set "under the rules of inference." In formal logic, the deductive closure of a theory is taken for granted—in fact, the formal definition of a "theory" is a set of assumptions closed under the rules of inference (Tarski 1956). In reality, however, it is impossible to generate the complete deductive closure of a premise set, for the resulting set is infinite (since it contains all *tautologies*, i.e., expressions that are always true and hence follow from any set). In consequence, the complete deductive closure of a premise set is neither realizable nor desirable—only *partial* deductive closures provide useful results.

In this paper, we present an algorithm that performs an efficient partial deductive closure for an important class of formulas. We call this class SPtSP, ("single property to single property"); it comprises conditional formulas that link one property of an object to another property of an object. Examples include: *if the size of an organization increases, then its inertia increases*; and *if the inertia of an organization increases, then its survival chance increases*. Statements of this form provide the backbone of any empirical social science; arguably, it is the most important class of empirical statements in the social sciences.

In §2, we give an overview of the representation language, FOL. Section 3 discusses the role of formal logic in theory-building; in analogy to the cyclic *research process* in empirical research, we describe a *logical cycle* of theory development. Section 4 describes the algorithm. The next two sections (5 and 6) show the algorithm in action, working on the "inertia" fragment of an important organization theory, *Organizational Ecology* (Hannan and Freeman 1984 and 1989). As it turns out, our algorithm generates more theorems than the original discursive theory of organizational inertia made explicit, some of which are of theoretical interest. The last section discusses some limits to our approach, regarding both the algorithm and the use of FOL as a representation language.

The research reported in this article is part of a larger effort at the Applied Logic Laboratory (ALL) of the University of Amsterdam to develop a formal methodology of theory analysis and theory building. This effort includes the application of standard logics to existing theories in organization and management (Péli et al. 1994; Péli and Masuch 1997; Bruggeman 1997), the development of "nonstandard" logics especially suited for the representation of *action* theories (Huang et al. 1996, Masuch and Huang 1996, Pólos and Masuch 1995), and the development of software that supports theory building using formal logic (Ó Nualláin 1993).

2. Formal Machinery: First Order Logic

The formal language used in this article is first order logic (FOL; see Péli et al. 1994, Appendix A, or standard textbooks, such as Jeffrey 1967, Gamut 1991, Gabbay and Guenthner 1989). Developing from Aristotle's treatment of syllogisms, FOL is usually identified with "classical" logic, or even with *the* logic by the general public. FOL is based on the idea that the domain consists of objects (the "universe" of discourse) that have certain properties, or that stand in certain relations with each other. Objects are represented by symbols for constants and variables; properties and relations are referred to by predicate symbols. More specifically, there are:

Quantifiers: symbols that range over the domain, i.e., $\forall$ "for all," and $\exists$ "exists."

Variables: name slots for objects that allow the use of quantifiers (roughly comparable to pronouns in English, we use lower case symbols, e.g., p, q, x, x_1, $\cdots$).

Predicate constants: names for properties of, or relations between, objects (we use capitalized strings of symbols, e.g., $Size(o_1, s_1)$, or infix predicates, e.g., $(s_1 > s_2)$).

Logical connectives: symbols that allow the building of complex expressions from simple expressions (the five standard connectives are negation ($\neg$ "not"), disjunction ($\vee$ "or"), conjunction ($\wedge$ "and"), conditional ($\rightarrow$ "if-then"), and biconditional ($\leftrightarrow$ "if-and-only-if")).

A first order language may also include constants (fixed names for objects) and function symbols. Table 1 summarizes the logical symbols of first order logic, and Table 2 is a truth table for the logical connectives. Here are four examples of expressions in FOL. The first example formally expresses transitivity of ">"; the next three examples are from the formalization of Organizational Ecology (further explained in §5):

Table 1 **Logical Symbols of First Order Logic**

Quantifiers	$\forall$	For All
	$\exists$	Exists
Connectives	$\neg$	not
	$\vee$	or
	$\wedge$	and
	$\rightarrow$	if-then
	$\leftrightarrow$	if-and-only-if

• If x is larger than y and y is larger than z then x is larger than z:

$$\forall x, y, z((x > y) \wedge (y > z) \rightarrow (x > z)).$$

• For every organization, there is some size that it has:

$$\forall x, t(O(x, t) \rightarrow \exists s(\text{Size}(x, s, t))).$$

(Read: For all x, t if x is an organization at time t then there exists some size s that x has at time t.)

• If an organization exists at t_1 and at t_2 then this organization exists between t_1 and t_2:

$$\forall x, t, t_1, t_2(O(x, t_1) \wedge O(x, t_2) \wedge (t > t_1) \wedge (t_2 > t) \rightarrow O(x, t)).$$

(Read: For all x, t, t_1, t_2 if x is an organization at time t_1 and x is an organization at time t_2 and t is a timepoint between t_1 and t_2 ($t_1 < t < t_2$) then x is an organization at time t.)

• More complex organizations have longer reorganization periods than less complex organizations of the same class during similar reorganizations:

Table 2 **Truth Table for the Logical Connectives ("*T*" symbolizes *true*, "*F*" symbolizes *false*)**

p	q	$\neg p$	$p \vee q$	$p \wedge q$	$p \rightarrow q$	$p \leftrightarrow q$
F	F	T	F	F	T	T
F	T	T	T	F	T	F
T	F	F	T	F	F	F
T	T	F	T	T	T	T

$$\forall x, y, re, c, c_1, c_2, t_a, t_b, t_c(O(x, t_a) \wedge O(y, t_a) \wedge O(y, t_c) \wedge \text{Class}(x, c, t_a) \wedge \text{Class}(y, c, t_a) \wedge \text{Reorg}(x, t_a, t_b) \wedge \text{Reorg}(y, t_a, t_c) \wedge \text{Reorg\_type}(x, re, t_a) \wedge \text{Reorg\_type}(y, re, t_a) \wedge \text{Compl}(x, c_1, t_a) \wedge \text{Compl}(y, c_2, t_a) \wedge (c_2 > c_1) \rightarrow (t_c > t_b)).$$

(Read: For all x, y, re, c, c_1, c_2, t_a, t_b, t_c if x and y are organizations of the same class c beginning the same type of reorganization re at time t_a with complexities c_1 and c_2 respectively, and x finishes reorganization at t_b, and y finishes reorganization alive at time t_c, and c_2 exceeds c_1, then t_c exceeds t_b.)

Propositional systems can be represented in FOL as sets of formalized statements ordered according to the relation of logical consequence. Such an ordering distinguishes between premises and conclusions; conclusions are justified if they are derivable from premises through *sound* inference rules. Here is a classical example from Aristotle's collection of syllogisms (here *Socrates* is a term constant):

All men are mortal	$\forall x(\text{Man}(x) \rightarrow \text{Mortal}(x))$
Socrates is a man	Man(Socrates)
Socrates is mortal	Mortal(Socrates)

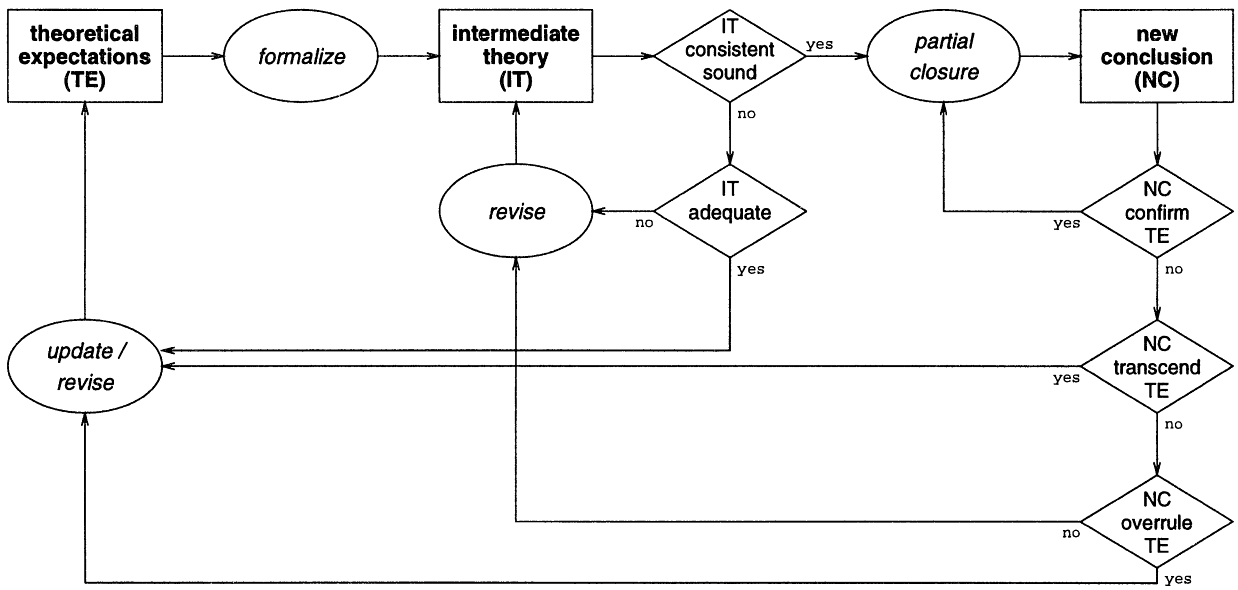
The first two statements are premises; the last statement is a conclusion. The next example gives an invalid syllogism:

Some men are mortal	$\exists x(\text{Man}(x) \wedge \text{Mortal}(x))$
Socrates is a man	Man(Socrates)
*Socrates is mortal	*Mortal(Socrates)

These premises do not justify the conclusion, since the existential statement about mortality does not exclude the possibility that some men are not mortal, and Socrates may be an instance of the immortal group.

From a logical point of view, a theoretical explanation coincides with *derivability*; the derivation "explains" the conclusion in logical terms (Salmon 1989). The notion derivability can be brought to bear upon the basic logical properties of a set of propositions; if the set is *inconsistent*, then it gives rise to contradictions, and hence to the derivation of the *falsum* (a formula that is always

Figure 1 The Logical Cycle



false). If the set is *tautological* (i.e., if the conjunction of its elements is always true) then its negation (i.e., the negation of the conjunction of its elements) gives rise to the *falsum*. If the set is *contingent* (falsifiable in the terms of standard epistemology), then neither the set itself nor its negation can give rise to the *falsum*; that is to say, it is neither necessarily true nor necessarily false.

3. Developing Theories with Logical Tools

Theories can be seen as propositional systems, and logic is traditionally used to order such systems. Logic provides the rationale for theoretical explanations; explanations, in turn, provide the justification for a theory. In this view, the "implications" of a theory are crucial for its justification: the better a theory, the better its "predictions". The formal definition of a theory is a set of statements "closed" under the rules of inference (Tarski 1956), which would suggest that the logic itself does the job of providing the implications. In practice, however, the logical closure of a theory is not a given; some agent is needed to make logic happen, and to carry out the deductions. As a consequence, we need to distinguish between a *premise set* (an explicitly stated set of premises), a *complete theory* (the premise set closed under the rules of inference), and *intermediate* theories that represent the premise set plus some, but not necessarily all, of its logical consequences. In addition, we must recognize that theories are not always conceived as explicitly stated sets of formulas; they can also be perceived as some kind of knowledge of a theoretician regarding a domain, regardless how this knowledge is represented. For want of a better term, we call this kind of knowledge *theoretical expectation*. Obviously, theoretical expectations depend on their bearer and may change as the theory evolves (Lakatos and Musgrave 1970).

Formal inferencing and theoretical expectations interact. Conclusions that confirm earlier expectations strengthen confidence in the theory and unearth new ways to test it. Unexpected conclusions invite the theoretician to revise either his expectations or the original theory. If the theory does not make the right predictions, it must change, so finding out which conclusions are implied by a theory is an important part of theory building. Much like the cyclic research process in empirical research (problem identification; hypothesis formulation; research design; data collection; data analysis; hypothesis testing), there is a *logical cycle* in the interaction between theoretical expectations and formal inferencing.

The logical cycle can be described using the terminology introduced above (Figure 1):

Theoretical expectations (TE) are formalized, yielding an intermediate theory (IT). The intermediate theory may or may not be consistent. If it is inconsistent, either the formalization or the theoretical expectations require revisions. If it is consistent, continued theorizing yields a partial closure of the original IT. New conclusions (NC) can (1) confirm the theoretical expectations, (2) transcend the theoretical expectations, or (3) contradict the theoretical expectation. In the first case, the partial closure can continue; in the second case, the theoretical expectations need to be updated; in the third case, either the formalization of the original expectations or the expectations themselves need to be revised. The cycle is never truly complete, since the deductive closure of any set is infinite. But not all conclusions that are technically derivable from a given set are of interest. For example, tautologies are derivable from any set of premises. But because they are always true, they tell us nothing about the domain—they are true regardless of the structure of the domain.

Although everybody agrees that theoretical labor is an important part of a researcher's work—thinking a theory through, taking it to its logical conclusions, ascertaining its consistency and coherence, and so on—the logical cycle has received very little attention in the literature. In the past, there was no method to guide the researcher more systematically through a maze of potential conclusions. With the advent of logical programming, however, we can now automate a partial deductive closure of a given set of assumptions.

4. Deduction of Theorems

We can get a partial deductive closure of a set of premises by applying inference rules to deduce new expressions (in technical terms "well-formed formulas," or "formulas" for short). In this section, we describe an algorithm that performs a partial deductive closure of an important class of theorems. Our algorithm focuses on theorems that relate two properties of the domain. We call this class of statements "single property to single property" (SPtSP); it comprises conditional formulas that link one property of an object to another property of an object, as noted above. The algorithm is called PDC-1.

4.1. Informal Description of the Algorithm

The algorithm uses premises of the form SPtSP. An SPtSP expression relates one quantifiable property to another, such as *higher inertia yields to higher survival chances*. Additionally, such an expression may have constraints that restrict it to certain types of objects, e.g., to *reorganization-free organizations*: *reorganization-free organizations with higher inertia have higher survival chances*. Assume that a domain is characterized by the following premises:

$$\text{Constraints}_1 \wedge \text{Property}_1 \rightarrow \text{Property}_2,$$

$$\text{Constraints}_2 \wedge \text{Property}_2 \rightarrow \text{Property}_3,$$

$$\text{Constraints}_3 \wedge \text{Property}_3 \rightarrow \text{Property}_4.$$

A theorem that relates Property_1 with Property_4 can be deduced from these three premises by "cutting out" Property_2 and Property_3:

$$\text{Constraints}_1 \wedge \text{Constraints}_2 \wedge \text{Constraints}_3 \wedge \text{Property}_1 \rightarrow \text{Property}_4.$$

As cases in point, we take three premises from the domain of Organizational Ecology. The following notation is used to represent SPtSP class expressions: $[\text{Constraints}] \wedge \text{Property}_1 \rightarrow \text{Property}_2$. The Property_1 and Property_2 are referring to two quantifiable properties of the domain. The [Constraints] are the restricting conjuncts (either one or more conjuncts). The square brackets "$[\cdots]$" are used to differentiate the conjuncts of the Constraints from Property_1 in the antecedent. These brackets are added only to improve readability, and are ignored by the formal machinery.

Assume we have the following premises (Assumptions 5, 3b, and 2a in §5, respectively):

• Larger organizations have higher inertia than smaller organizations of the same class:

$$\forall c, i_1, i_2, s_1, s_2, t_1, t_2, x, y([O(x, t_1) \wedge O(y, t_2) \wedge \text{Class}(x, c, t_1) \wedge \text{Class}(y, c, t_2) \wedge \text{Size}(x, s_1, t_1) \wedge \text{Size}(y, s_2, t_2) \wedge \text{Iner}(x, i_1, t_1) \wedge \text{Iner}(y, i_2, t_2)] \wedge (s_2 > s_1) \rightarrow (i_2 > i_1)).$$

(Read: For all c, i_1, i_2, s_1, s_2, t_1, t_2, x, y if x and y are organizations of the same class c at time t_1 and t_2, and s_1 and i_1 are, respectively, the size and inertia of x at t_1 and s_2 and i_2 are, respectively, the size and inertia of y at t_2, and s_2 exceeds s_1, then i_2 exceeds i_1.)

• Reorganization-free organizations with higher inertia have higher reproducibility:

$$\forall i_1, i_2, rp_1, rp_2, t_1, t_2, x, y([O(x, t_1) \wedge O(y, t_2) \wedge \text{Reorg\_free}(x, t_1, t_1) \wedge \text{Reorg\_free}(y, t_2, t_2) \wedge \text{Iner}(x, i_1, t_1) \wedge \text{Iner}(y, i_2, t_2) \wedge \text{Repr}(x, rp_1, t_1) \wedge \text{Repr}(y, rp_2, t_2)] \wedge (i_2 > i_1) \rightarrow (rp_2 > rp_1)).$$

(Read: For all i_1, i_2, rp_1, rp_2, t_1, t_2, x, y if x and y are organizations not in reorganization at time t_1 and t_2 respectively, and i_1 and rp_1 are, respectively, the inertia and reproducibility of x at t_1, and i_2 and rp_2 are, respectively, the inertia and reproducibility of y at t_2, and i_2 exceeds i_1 then rp_2 exceeds rp_1.)

• Organizations with higher reproducibility have higher reliability/accountability:

$$\forall ra_1, ra_2, rp_1, rp_2, t_1, t_2, x, y([O(x, t_1) \wedge O(y, t_2) \wedge \text{Repr}(x, rp_1, t_1) \wedge \text{Repr}(y, rp_2, t_2) \wedge \text{Relacc}(x, ra_1, t_1) \wedge \text{Relacc}(y, ra_2, t_2)] \wedge (rp_2 > rp_1) \rightarrow (ra_2 > ra_1)).$$

(Read: For all ra_1, ra_2, rp_1, rp_2, t_1, t_2, x, y if x and y are organizations at time t_1 and t_2 respectively, and rp_1 and ra_1 are, respectively, the reproducibility and reliability/accountability of x at t_1, and rp_2 and ra_2 are, respectively, the reproducibility and reliability/accountability of y at t_2, and rp_2 exceeds rp_1 then ra_2 exceeds ra_1.)

These premises are used to create a new theorem relating *size* and *reliability/accountability*, by following the same steps as in the abstract example. The set of constraints on this theorem is the superset of the constraints on the individual premises. Identical conjuncts in this set can be removed (since $(p \wedge p) \leftrightarrow p$). This leads to the following new theorem:

• Larger reorganization-free organizations have higher reliability/accountability than smaller organizations of the same class:

$$\forall c, ra_1, ra_2, s_1, s_2, t_1, t_2, x, y([O(x, t_1) \wedge O(y, t_2) \wedge \text{Reorg\_free}(x, t_1, t_1) \wedge \text{Reorg\_free}(y, t_2, t_2) \wedge \text{Class}(x, c, t_1) \wedge \text{Class}(y, c, t_2) \wedge \text{Size}(x, s_1, t_1) \wedge \text{Size}(y, s_2, t_2) \wedge \text{Relacc}(x, ra_1, t_1) \wedge \text{Relacc}(y, ra_2, t_2)] \wedge (s_2 > s_1) \rightarrow (ra_2 > ra_1)).$$

(Read: For all c, ra_1, ra_2, s_1, s_2, t_1, t_2, x, y if x and y are organizations of the same class c and not in reorganizational period at time points t_1 and t_2 respectively, and s_1 and ra_1 are, respectively, the size and reliability/accountability of x at t_1, and s_2 and ra_2 are, respectively, the size and reliability/accountability of y at t_2, and s_2 exceeds s_1 then ra_2 exceeds ra_1.)

4.2. Formal Specification

The partial deductive closure of a premise set is generated in three steps:

Filter premises: The premises of the domain are filtered for SPtSP premises. Only these premises are used to construct new theorems.

Deduce new theorems: The construction of new theorems. The algorithm uses the SPtSP premises to derive new SPtSP theorems.

Filter new theorems: The set of constructed theorems is refined by removing: (1) vacuously true theorems, (2) weaker versions of other theorems in the set, and (3) superfluous conjuncts.

Table 3 introduces the notation we use to refer to the premise set and the derived theorems.

4.2.1. Filter Premises. The algorithm PDC-1 first identifies the premises that are in SPtSP form, that is, of the form:

$$[\text{Constraints}_1] \wedge \text{Property}_1 \rightarrow \text{Property}_2.$$

Table 3 **Premise Sets and Theorems**

Symbol	Description
Σ	The original premise set used as a starting point for the algorithm
Σ^*	The original premise set Σ restricted to SPtSP premises
Σ'	The set of SPtSP theorems derived from Σ^*
Σ^+	The resulting set of premises and theorems $\Sigma \cup \Sigma'$, i.e., a partial closure of Σ

Let Σ^* denote the premise set Σ restricted to SPtSP premises. Define Σ^* as follows:

1. For every premise $\sigma \in \Sigma$:
   (a) IF σ belongs to the SPtSP class
   THEN $\sigma \in \Sigma^*$
   ELSE $\sigma \notin \Sigma^*$
2. Nothing else is in Σ^*

4.2.2. Deduce New Theorems. PDC-1 constructs new theorems for each pair of properties described in the SPtSP premises. The algorithm uses a depth-first, forward-chaining strategy.

Let Σ^* denote the set of SPtSP premises and Φ the set of properties described in Σ^*. Let σ_i denote an SPtSP formula ([Constr_i] $\wedge$ $\text{Prop}_a \rightarrow \text{Prop}_i$). The deduced theorems are denoted by Σ', the set of SPtSP theorems derived from Σ^*.

Step 1. The algorithm constructs new theorems for each pair $\{\text{Prop}_a, \text{Prop}_b\}$ of properties in Φ. Pairs of the same properties $\{\text{Prop}_a, \text{Prop}_a\}$ are excluded, since they would yield tautologies like *organizations with higher inertia have higher inertia*.

Step 2. For each pair of properties $\{\text{Prop}_a, \text{Prop}_b\}$ the algorithm uses a premise having Prop_a in the antecedent:

$$[\text{Constr}_1] \wedge \text{Prop}_a \rightarrow \text{Prop}_1,$$

to construct an initial formula σ_1, which is refined step by step. If there are no (more) premises that relate Prop_a to another property, there are no (more) theorems that relate Prop_a and Prop_b. The algorithm terminates for $\{\text{Prop}_a, \text{Prop}_b\}$ and continues with the next pair of properties.

Step 3. The formula σ_i that relates Prop_a and Prop_i

$$[\text{Constr}_i] \wedge \text{Prop}_a \rightarrow \text{Prop}_i,$$

is refined if the algorithm can find a premise that relates Prop_i and Prop_{i+1}:

$$[\text{Constr}_{i+1}] \wedge \text{Prop}_i \rightarrow \text{Prop}_{i+1}.$$

This results in a formula σ_{i+1} that relates Prop_i and Prop_{i+1}:

$$[\text{Constr}_i \wedge \text{Constr}_{i+1}] \wedge \text{Prop}_a \rightarrow \text{Prop}_{i+1}.$$

To avoid cycles in this refinement process, the algorithm only considers premises that introduce a new property. For example, if an initial formula relating *inertia* and *reproducibility* is found in Step 2, then a premise relating *reproducibility* and *inertia* is not applied in Step 3, since the property *inertia* was used before. This prevents the construction of some tautologies (like *organizations with higher inertia have higher inertia*).

If the formula σ_i cannot be further refined (there is no (further) premise that relates Prop_i and Prop_{i+1}), the algorithm has reached a dead end. The algorithm retracts the last refinement σ_i and attempts to construct other refinements of σ_{i-1} (the previous version of σ_i).

Step 4. The refinement of formula σ_i is completed if a theorem is constructed that relates Prop_a and Prop_b (in other words Prop_{i+1} in Step 3 is the desired Prop_b). The new theorem σ_{i+1} is added to Σ', the set of deduced SPtSP theorems. Since there may be more than one theorem that relates Prop_a and Prop_b, the algorithm retracts σ_{i+1} after a successful theorem construction in Step 4 in order to find formulas constructed using different combinations of premises. This allows for the construction of theorems relating to different contexts, like *the survival chance of reorganizing organizations decreases with time* and *the survival chance of reorganization-free organizations increases with time*.

If Prop_{i+1} in Step 3 is not the desired Prop_b, Step 3 is repeated.

PDC-1 uses a standard algorithm for variable unification (Robinson 1965) in Step 3. This variable unification is necessary to determine that a formula, such as $\forall o_1, t_a(O(o_1, t_a))$, can be made equal to another formula, such as $\forall x, y(O(x, y))$, because the variables can be unified, in this case x with o_1 and y with t_a.

4.2.3. Filter Theorems. We have three filters to refine the set of constructed theorems. The first filter removes vacuously true theorems, the second removes weaker or identical versions of theorems, and the last filter removes superfluous conjuncts in theorems. The filters create a more concise set of theorems by removing non-interesting theorems.

Vacuously True Theorems. Sometimes premises cannot be combined directly, since different premises may have incompatible constraints. If we construct a new candidate theorem with incompatible premises, i.e., premises having contradictory constraints, for example, we create a vacuously true theorem by combining premises for reorganizing organizations with premises for reorganization-free organizations. A vacuously true theorem has an antecedent that can never be fulfilled in the context of the premises (there is no model that satisfies both the antecedent and the premises). Recall that the theorems have the form of an *if-then* statement: if the antecedent holds then the consequent must hold. Therefore, we can prove any consequent from the theorem's antecedent if the antecedent never holds. In the truth table for the logical conditional, Table 2, if p is false then the conditional $p \rightarrow q$ is always true, regardless of q's truth value.

Suppose that the premise set contains the following premise:

An organization cannot be reorganization-free and reorganizing at the same time:

$$\forall x, t_1, t_2(\neg(\text{Reorg\_free}(x, t_1, t_2) \wedge \text{Reorg}(x, t_1, t_2))).$$

Assume that we have constructed a theorem with the following constraints:

For all reorganization-free organizations under reorganization . . .

$$\forall x, t_1, t_2, \cdots([O(x, t_1) \wedge \text{Reorg\_free}(x, t_1, t_2) \wedge \text{Reorg}(x, t_1, t_2) \wedge \cdots] \wedge \cdots \rightarrow \cdots).$$

In this case, the antecedent of the theorem is inconsistent with the premise set, and the theorem is true regardless of the consequent (which may be even the *falsum*).

In sum, if we can substitute the *falsum* for the consequent in a theorem, and this theorem still holds, then the theorem is vacuously true. Such a theorem only holds because of the (hidden) contradiction in its antecedent. Vacuously true theorems do not provide any theoretical insights, therefore, their removal does not affect the theory.

Filter Out Weaker Versions of a Theorem. Different premises can lead to different theorems that relate the same pair of properties. Sometimes these theorems are complementary: for example, one theorem is restricted to reorganizations, and another to reorganization-free periods, like *the survival chance of reorganizing organizations decreases with time* and *the survival chance of reorganization-free organizations increases with time*. But in other cases, one of them may subsume the other: for example, if one theorem states that *the inertia of organization x is larger than the inertia of organization y* and another theorem that *the inertia of organization x is larger than* **or equal to** *the inertia of organization y*, the latter theorem is weaker. In this case weaker versions of the same theorem are removed. The filter evaluates for every theorem whether it is the unique or strongest version of the formula. Identical theorems and theorems with weaker antecedents or stronger consequents are removed. Identical or weaker versions of a theorem are uninteresting because they can be derived from a stronger version. For example,

$$[\text{Org}(o_1) \wedge \text{Org}(o_2)] \wedge (\text{Prop}_1(o_1) \geq \text{Prop}_1(o_2)) \rightarrow (\text{Prop}_2(o_1) > \text{Prop}_2(o_2)),$$

is preferable to

$$[\text{Org}(o_1) \wedge \text{Org}(o_2) \wedge \text{Reorg}(o_1) \wedge \text{Reorg}(o_2)] \wedge (\text{Prop}_1(o_1) > \text{Prop}_1(o_2)) \rightarrow (\text{Prop}_2(o_1) \geq \text{Prop}_2(o_2)),$$

since $[\text{Org}(o_1) \wedge \text{Org}(o_2)]$ is implied by $[\text{Org}(o_1) \wedge \text{Org}(o_2) \wedge \text{Reorg}(o_1) \wedge \text{Reorg}(o_2)]$; and $(\text{Prop}_1(o_1) \geq \text{Prop}_1(o_2))$ is implied by $(\text{Prop}_1(o_1) > \text{Prop}_1(o_2))$; and finally $(\text{Prop}_2(o_1) > \text{Prop}_2(o_2))$ implies $(\text{Prop}_2(o_1) \geq \text{Prop}_2(o_2))$.

Simplify Theorems. A constructed theorem may require the existence of "intermediate" conjuncts. The formula relating *size* and *reliability/accountability* derived in the example of §4.1 would actually read:

$$\begin{aligned}&\forall c, i_1, i_2, ra_1, ra_2, rp_1, rp_2, s_1, s_2, t_1, t_2, x, y([O(x, t_1)\\ &\wedge O(y, t_2) \wedge \text{Reorg\_free}(x, t_1, t_1)\\ &\wedge \text{Reorg\_free}(y, t_2, t_2) \wedge \text{Class}(x, c, t_1)\\ &\wedge \text{Class}(y, c, t_2) \wedge \text{Size}(x, s_1, t_1) \wedge \text{Size}(y, s_2, t_2)\\ &\wedge \mathbf{Iner(x, i_1, t_1)} \wedge \mathbf{Iner(y, i_2, t_2)}\\ &\wedge \mathbf{Repr(x, rp_1, t_1)} \wedge \mathbf{Repr(y, rp_2, t_2)}\\ &\wedge \text{Relacc}(x, ra_1, t_1) \wedge \text{Relacc}(y, ra_2, t_2)]\\ &\wedge (s_2 > s_1) \rightarrow (ra_2 > ra_1)).\end{aligned}$$

The *inertia* and *reproducibility* conjuncts must exist for the conditions of the premises to be fulfilled. If their existence is postulated (as is the case in the formal inertia theory), these conjuncts can be removed from the constraints.

- For every organization, there is some inertia that it has:

$$\forall x, t(O(x, t) \rightarrow \exists i(\text{Iner}(x, i, t))).$$

- For every organization, there is some reproducibility that it has:

$$\forall x, t(O(x, t) \rightarrow \exists rp(\text{Repr}(x, rp, t))).$$

The *inertia* and *reproducibility* conjuncts can now be derived from the *organization* conjuncts. The set of constraints is simplified (this was tacitly done in §4.1):

$$\begin{aligned}&\forall c, ra_1, ra_2, s_1, s_2, t_1, t_2, x, y([O(x, t_1)\\ &\quad \wedge O(y, t_2) \wedge \text{Reorg\_free}(x, t_1, t_1)\\ &\quad \wedge \text{Reorg\_free}(y, t_2, t_2) \wedge \text{Class}(x, c, t_1)\\ &\quad \wedge \text{Class}(y, c, t_2) \wedge \text{Size}(x, s_1, t_1) \wedge \text{Size}(y, s_2, t_2)\\ &\quad \wedge \text{Relacc}(x, ra_1, t_1) \wedge \text{Relacc}(y, ra_2, t_2)]\\ &\quad \wedge (s_2 > s_1) \rightarrow (ra_2 > ra_1)).\end{aligned}$$

This concludes the description of our algorithm for the partial closure of SPtSP formulas. In the next two sections, we show the algorithm in action; it is applied to a fragment of an important organization theory, the "inertia" part of Organizational Ecology (OE). We first give a brief account of OE, then provide a formalization of the inertia part of OE, and finally show how the algorithm performs the partial deductive closure of this part.

5. Inertia Fragment of Organizational Ecology

Most organizational theories regard organizations as agents that adapt rationally to changing environments (Thompson 1967, Mintzberg 1979). OE, in contrast, sees organizational structures evolving through environmental selection. When environmental conditions change, new organizations emerge, and maladapted organizations die.

Organizational ecology employs analogies from biology. Genes determine the action repertoire of organisms, whereas organizations' repertoires are fixed by their core features. Organizations of the same form make up a population (just as organisms of the same form make up a species). Several factors inhibit the flexibility and adaptation of organizations, such as sunk costs, political coalitions, or the hazards of lost legitimacy.

Organizational ecology considers changes in the environment to be largely unpredictable. Organizations are characterized by structural inertia—if they adapt, they do so slowly. Contrary to the rational adaptation approach, however, successful organizations are inert, not flexible. Organizations must produce their products or services reliably and account for their actions rationally. To do so, they must be able to reproduce their structures smoothly. But the factors that facilitate their reproducibility make organizations resistant to change. Thus, inertia is a byproduct of reproducibility.

Organizational ecology does recognize the possibility of rational adaptation. To adapt, organizations must reorganize. Organizations can change their structures to a certain degree, but reorganizations typically involve changes in core features. Such changes are dangerous; they involve large resources, and organizational learning must start anew and higher up on the learning curve. Even if an organization survives a major reorganization, its environment may change in unexpected ways and the reorganization might be in vain. So if organizations attempt to adapt, they are not likely to succeed. Inert organizations—those that resist the temptation to reorganize—may be less at risk than flexible organizations.

The inertia part of OE is originally described in Hannan and Freeman (1984). A formalization of this theory in FOL has been published in Péli et al. (1994). We use the formulas representing the premises of Péli et al. (1994) as input, and let our algorithm derive the theorems. The premises will be discussed in this section, and the derivable theorems in §6. Table 4 characterizes the relation symbols that are used in the formulas of the inertia fragment.

As noted above, OE stipulates that inertia—not flexibility—helps organizations to survive (Theorem 1), the reason being that inertia is associated with reliability and other features that help organizations to survive. Assumptions 1–3 are put forward to justify Theorem 1

Table 4 **The Meaning of the Relation Symbols**

Symbol	Description
Class(x,c,t)	Object x is a member of class c at time t
Comp(x,cp,t)	Object x is characterized by complexity cp at time t
Iner(x,i,t)	Object x has a value of inertia i at time t
$O(x,t)$	Object x is an organization at time t
Relacc(x,ra,t)	Object x has a value of reliability/accountability ra at time t
Reorg(x, t_1, t_2)	Object x reorganizes between times t_1 and t_2
Reorg_free(x,t_1,t_2)	Object x has a reorganization-free period between times t_1 and t_2
Reorg_type(x,rt,t)	Object t is in a reorganization of type rt at time t
Repr(x,rp,t)	Object x has a value of reproducibility rp at time t
Sc(x,p,t)	The chance of survival of object x is p at time t
Size(x,s,t)	Object x has a size s at time t
Time(t)	Object t is a time-point
$x > y$	Value x is larger than value y

(the original justification in Hannan and Freeman 1984 is not sound, but one can derive the theorem by strengthening the assumptions, as shown in Péli et al. 1994).

ASSUMPTION 1. *Organizations with higher reliability and accountability have higher survival chance*:

$$\forall p_1, p_2, ra_1, ra_2, t_1, t_2, x, y([O(x, t_1) \wedge O(y, t_2) \wedge \text{Relacc}(x, ra_1, t_1) \wedge \text{Relacc}(y, ra_2, t_2) \wedge \text{Sc}(x, p_1, t_1) \wedge \text{Sc}(y, p_2, t_2)] \wedge (ra_2 > ra_1) \rightarrow (p_2 > p_1)).$$

ASSUMPTION 2a. *Organizations with higher reproducibility have higher reliability and accountability*:

$$\forall ra_1, ra_2, rp_1, rp_2, t_1, t_2, x, y([O(x, t_1) \wedge O(y, t_2) \wedge \text{Relacc}(x, ra_1, t_1) \wedge \text{Relacc}(y, ra_2, t_2) \wedge \text{Repr}(x, rp_1, t_1) \wedge \text{Repr}(y, rp_2, t_2)] \wedge (rp_2 > rp_1) \rightarrow (ra_2 > ra_1)).$$

ASSUMPTION 2b. *Organizations with higher reliability and accountability have higher reproducibility*:

$$\forall ra_1, ra_2, rp_1, rp_2, t_1, t_2, x, y([O(x, t_1) \wedge O(y, t_2) \wedge \text{Relacc}(x, ra_1, t_1) \wedge \text{Relacc}(y, ra_2, t_2) \wedge \text{Repr}(x, rp_1, t_1) \wedge \text{Repr}(y, rp_2, t_2)] \wedge (ra_2 > ra_1) \rightarrow (rp_2 > rp_1)).$$

ASSUMPTION 3a. *Reorganization-free organizations with higher reproducibility have higher inertia*:

$$\forall i_1, i_2, rp_1, rp_2, t_1, t_2, x, y([O(x, t_1) \wedge O(y, t_2) \wedge \text{Reorg\_free}(x, t_1, t_1) \wedge \text{Reorg\_free}(y, t_2, t_2) \wedge \text{Repr}(x, rp_1, t_1) \wedge \text{Repr}(y, rp_2, t_2) \wedge \text{Iner}(x, i_1, t_1) \wedge \text{Iner}(y, i_2, t_2)] \wedge (rp_2 > rp_1) \rightarrow (i_2 > i_1)).$$

ASSUMPTION 3b. *Reorganization-free organizations with higher inertia have higher reproducibility*:

$$\forall i_1, i_2, rp_1, rp_2, t_1, t_2, x, y([O(x, t_1) \wedge O(y, t_2) \wedge \text{Reorg\_free}(x, t_1, t_1) \wedge \text{Reorg\_free}(y, t_2, t_2) \wedge \text{Repr}(x, rp_1, t_1) \wedge \text{Repr}(y, rp_2, t_2) \wedge \text{Iner}(x, i_1, t_1) \wedge \text{Iner}(y, i_2, t_2)] \wedge (i_2 > i_1) \rightarrow (rp_2 > rp_1)).$$

The next two theorems spell out the consequences of environmental selection through time: (surviving) organizations will tend to become increasingly inert (Theorem 2), so their survival chances increase (Theorem 3). Justifying these theorems requires the assumption that "reproducibility of structure increases monotonically with age" (Assumption 4).

ASSUMPTION 4. *The reproducibility of reorganization-free organizations increases with time*:

$$\forall rp_1, rp_2, t_1, t_2, x([\text{Time}(t_1) \wedge \text{Time}(t_2) \wedge O(x, t_1) \wedge O(x, t_2) \wedge \text{Reorg\_free}(x, t_1, t_2) \wedge \text{Repr}(x, rp_1, t_1) \wedge \text{Repr}(x, rp_2, t_2)] \wedge (t_2 > t_1) \rightarrow (rp_2 > rp_1)).$$

Under the heading of "reorganization," Theorem 4 covers organizational change: organizations may attempt structural change, but such change puts the organization more at risk than inertia. Hannan and Freeman (1984) claim that Theorem 4 relies on the assumptions that reorganization lowers the reliability of organizational performance (Assumption 6), and that the structural inertia of an organization increases with size (for organizations belonging to the same class; Assumption 5).

ASSUMPTION 5. *Larger organizations of the same class have higher inertia*:

$$\forall c, i_1, i_2, s_1, s_2, t_1, t_2, x, y([O(x, t_1) \wedge O(y, t_2) \wedge \text{Class}(x, c, t_1) \wedge \text{Class}(y, c, t_2) \wedge \text{Size}(x, s_1, t_1) \wedge \text{Size}(y, s_2, t_2) \wedge \text{Iner}(x, i_1, t_1) \wedge \text{Iner}(y, i_2, t_2)] \wedge (s_2 > s_1) \rightarrow (i_2 > i_1)).$$

ASSUMPTION 6. *The reliability and accountability of reorganizing organizations decreases with time*:

$$\forall ra_1, ra_2, t_1, t_2, x([\text{Time}(t_1) \wedge \text{Time}(t_2) \wedge O(x, t_1) \wedge O(x, t_2) \wedge \text{Reorg}(x, t_1, t_2) \wedge \text{Relacc}(x, ra_1, t_1) \wedge \text{Relacc}(x, ra_2, t_2)] \wedge (t_2 > t_1) \rightarrow (ra_1 > ra_2)).$$

The last and fifth theorem of the inertia fragment states that "complexity increases the risk of death due to reorganization." To simplify the setup, the fifth theorem is added as a premise (Assumption 7); this gives a total of seven assumptions.[1]

ASSUMPTION 7 (Theorem 5). *More complex organizations of the same class have lower survival chances after reorganizations of the same type*:

$$\forall c, c_1, c_2, p, p_1, p_2, re, t_a, t_b, t_c, x, y([O(x, t_a) \wedge O(y, t_a) \wedge O(x, t_c) \wedge O(y, t_c) \wedge \text{Class}(x, c, t_a) \wedge \text{Class}(y, c, t_a) \wedge \text{Sc}(x, p, t_a) \wedge \text{Sc}(y, p, t_a) \wedge \text{Reorg}(x, t_a, t_b) \wedge \text{Reorg}(y, t_a, t_c) \wedge \text{Reorg\_type}(x, re, t_a) \wedge \text{Reorg\_type}(y, re, t_a) \wedge \text{Reorg\_free}(x, t_b, t_c) \wedge \text{Sc}(x, p_1, t_c) \wedge \text{Sc}(y, p_2, t_c) \wedge \text{Compl}(x, c_1, t_a) \wedge \text{Compl}(y, c_2, t_a)] \wedge (c_2 > c_1) \rightarrow (p_1 > p_2)).$$

[1] The fifth theorem is a kind of meta-theorem. It takes as an assumption that *complexity causes longer reorganization periods*. Due to the fact that survival chance increases during reorganization-free periods (Theorems 3) and decreases during reorganization (Theorem 4), it concludes that complexity decreases the survival chance due to reorganization. This meta-reasoning requires a slightly more complex form of the algorithm than the version presented in this article. We treat the original fifth theorem as an assumption (Assumption 7), which enables us to use the simpler version of the PDC-1 algorithm and still derive the same number of theorems.

In addition, the premise set contains twelve premises that formulate the necessary background knowledge, e.g., organizations are either reorganizing or not reorganizing. These premises are implicitly used in the original text (Meaning Postulates 1–12).

MEANING POSTULATE 1. *Reorganization-free from t_1 to t_2 means reorganization-free at t_1 and at t_2*:

$$\forall x, t_1, t_2(\text{Reorg\_free}(x, t_1, t_2) \rightarrow \text{Reorg\_free}(x, t_1, t_1) \wedge \text{Reorg\_free}(x, t_2, t_2)).$$

MEANING POSTULATE 2. *Something that is equal cannot be larger, and something that is larger cannot be smaller*:

$$\forall x, y(\neg((x > y) \wedge (x = y)) \wedge \neg((x > y) \wedge (y > x))).$$

MEANING POSTULATE 3. *If x is larger than y and y is larger than z, then x is larger than z*:

$$\forall x, y, z((x > y) \wedge (y > z) \rightarrow (x > z)).$$

MEANING POSTULATE 4. *A reorganization takes time*:

$$\forall x, t_a, t_b(\text{Reorg}(x, t_a, t_b) \rightarrow (t_b > t_a)).$$

MEANING POSTULATE 5. *An organization cannot be reorganization-free and reorganizing at the same time*:

$$\forall x, t_1, t_2(\neg(\text{Reorg\_free}(x, t_1, t_2) \wedge \text{Reorg}(x, t_1, t_2))).$$

MEANING POSTULATE 6. *An organization cannot change its class without reorganizing*:

$$\forall x, t_1, t_2, c_1, c_2(O(x, t_1) \wedge O(x, t_2) \wedge \text{Reorg\_free}(x, t_1, t_2) \wedge \text{Class}(x, c_1, t_1) \wedge \text{Class}(x, c_2, t_2) \rightarrow (c_1 = c_2)).$$

MEANING POSTULATE 7. *If an organization exists at t_1 and at t_2, then this organization exists between t_1 and t_2*:

$$\forall x, t, t_1, t_2(O(x, t_1) \wedge O(x, t_2) \wedge (t > t_1) \wedge (t_2 > t) \rightarrow O(x, t)).$$

MEANING POSTULATE 8. *For every organization, there is some reliability and accountability that it has*:

$$\forall x, t(O(x, t) \rightarrow \exists ra(\text{Relacc}(x, ra, t))).$$

MEANING POSTULATE 9. *For every organization, there is some reproducibility that it has*:

$$\forall x, t(O(x, t) \rightarrow \exists rp(\text{Repr}(x, rp, t))).$$

MEANING POSTULATE 10. *For every organization, there is some survival chance that it has*:

$$\forall x, t(O(x, t) \rightarrow \exists p(\mathrm{Sc}(x, p, t))).$$

MEANING POSTULATE 11. *For every organization, there is some inertia that it has:*

$$\forall x, t(O(x, t) \rightarrow \exists i(\mathrm{Iner}(x, i, t))).$$

MEANING POSTULATE 12. *For every organization, there is some class to which it belongs*:

$$\forall x, t(O(x, t) \rightarrow \exists c(\mathrm{Class}(x, c, t))).$$

6. Application of PDC-1 to the Inertia Fragment

This section exemplifies the results of the PDC-1 when applied to the inertia part of OE. Starting with the seven assumptions and twelve background assumptions listed in §5, PDC-1 generates a total of seventeen theorems—twelve more than are presented in the original text. Several of the new theorems are theoretically important. The first five theorems coincide with the theorems of the original text; their theoretical importance has been justified in Hannan and Freeman (1984).

THEOREM 1. *Reorganization-free organizations with higher inertia have higher survival chances*:

$$\forall i_1, i_2, p_1, p_2, t_1, t_2, x, y([\mathrm{Reorg\_free}(x, t_1, t_1) \land \mathrm{Reorg\_free}(y, t_2, t_2) \land \mathrm{Iner}(x, i_1, t_1) \land \mathrm{Iner}(y, i_2, t_2) \land O(x, t_1) \land O(y, t_2) \land \mathrm{Sc}(x, p_1, t_1) \land \mathrm{Sc}(y, p_2, t_2)] \land (i_2 > i_1) \rightarrow (p_2 > p_1)).$$

THEOREM 2. *The inertia of reorganization-free organizations increases with time*:

$$\forall i_1, i_2, t_1, t_2, x([\mathrm{Time}(t_1) \land \mathrm{Time}(t_2) \land \mathrm{Reorg\_free}(x, t_1, t_2) \land O(x, t_1) \land O(x, t_2) \land \mathrm{Reorg\_free}(x, t_1, t_1) \land \mathrm{Reorg\_free}(x, t_2, t_2) \land \mathrm{Iner}(x, i_1, t_1) \land \mathrm{Iner}(x, i_2, t_2)] \land (t_2 > t_1) \rightarrow (i_2 > i_1)).$$

THEOREM 3. *The survival chance of reorganization-free organizations increases with time*:

$$\forall p_1, p_2, t_1, t_2, y([\mathrm{Sc}(y, p_1, t_1) \land \mathrm{Sc}(y, p_2, t_2) \land \mathrm{Time}(t_1) \land \mathrm{Time}(t_2) \land O(y, t_1) \land O(y, t_2) \land \mathrm{Reorg\_free}(y, t_1, t_2)] \land (t_2 > t_1) \rightarrow (p_2 > p_1)).$$

THEOREM 4. *The survival chance of reorganizing organizations decreases with time*:

$$\forall p_1, p_2, t_1, t_2, y([\mathrm{Sc}(y, p_1, t_1) \land \mathrm{Sc}(y, p_2, t_2) \land \mathrm{Time}(t_2) \land \mathrm{Time}(t_1) \land O(y, t_2) \land O(y, t_1) \land \mathrm{Reorg}(y, t_2, t_1)] \land (t_1 > t_2) \rightarrow (p_2 > p_1)).$$

THEOREM 5. **(Assumption 7.)** *More complex organizations of the same class have lower survival chances after reorganizations of the same type*:

$$\forall c, c_1, c_2, p, p_1, p_2, re, t_a, t_b, t_c, x, y([O(x, t_a) \land O(y, t_a) \land O(x, t_c) \land O(y, t_c) \land \mathrm{Class}(x, c, t_a) \land \mathrm{Class}(y, c, t_a) \land \mathrm{Sc}(x, p, t_a) \land \mathrm{Sc}(y, p, t_a) \land \mathrm{Reorg}(x, t_a, t_b) \land \mathrm{Reorg}(y, t_a, t_c) \land \mathrm{Reorg\_type}(x, re, t_a) \land \mathrm{Reorg\_type}(y, re, t_a) \land \mathrm{Reorg\_free}(x, t_b, t_c) \land \mathrm{Sc}(x, p_1, t_c) \land \mathrm{Sc}(y, p_2, t_c) \land \mathrm{Compl}(x, c_1, t_a) \land \mathrm{Compl}(y, c_2, t_a)] \land (c_2 > c_1) \rightarrow (p_1 > p_2)).$$

Theorems 6 and 7 are straightforward extensions of Assumptions 4 and 6.

THEOREM 6. *The reliability and accountability of reorganization-free organizations increases with time*:

$$\forall ra_1, ra_2, t_1, t_2, y([\mathrm{Relacc}(y, ra_1, t_1) \land \mathrm{Relacc}(y, ra_2, t_2) \land \mathrm{Time}(t_1) \land \mathrm{Time}(t_2) \land O(y, t_1) \land O(y, t_2) \land \mathrm{Reorg\_free}(y, t_1, t_2)] \land (t_2 > t_1) \rightarrow (ra_2 > ra_1)).$$

THEOREM 7. *The reproducibility of reorganizing organizations decreases with time*:

$$\forall rp_1, rp_2, t_1, t_2, x([\mathrm{Time}(t_1) \land \mathrm{Time}(t_2) \land \mathrm{Reorg}(x, t_1, t_2) \land O(x, t_2) \land O(x, t_1) \land \mathrm{Repr}(x, rp_1, t_2) \land \mathrm{Repr}(x, rp_2, t_1)] \land (t_2 > t_1) \rightarrow (rp_2 > rp_1)).$$

The first important new theorem, Theorem 8, says that organizational size has a positive impact on survival chance. It hinges on Assumption 5, and confirms the theoretical expectation regarding the context of environmental selection. It helps build confidence in the premise set as an adequate representation of OE.

THEOREM 8. *Larger reorganization-free organizations of the same class have higher survival chances*:

$$\forall c, p_1, p_2, s_1, s_2, t_1, t_2, x, y([\text{Class}(x, c, t_1) \wedge \text{Class}(y, c, t_2) \wedge \text{Size}(x, s_1, t_1) \wedge \text{Size}(y, s_2, t_2) \wedge \text{Reorg\_free}(x, t_1, t_1) \wedge \text{Reorg\_free}(y, t_2, t_2) \wedge O(x, t_1) \wedge O(y, t_2) \wedge \text{Sc}(x, p_1, t_1) \wedge \text{Sc}(y, p_2, t_2)] \wedge (s_2 > s_1) \rightarrow (p_2 > p_1)).$$

Theorems 9 and 10 follow from Theorem 8 on the basis of the reasoning leading to Theorem 1. They are extensions of Theorem 8.

THEOREM 9. *Larger reorganization-free organizations of the same class have higher reproducibility*:

$$\forall c, rp_1, rp_2, s_1, s_2, t_1, t_2, x, y([\text{Reorg\_free}(x, t_1, t_1) \wedge \text{Reorg\_free}(y, t_2, t_2) \wedge \text{Repr}(x, rp_1, t_1) \wedge \text{Repr}(y, rp_2, t_2) \wedge O(x, t_1) \wedge O(y, t_2) \wedge \text{Class}(x, c, t_1) \wedge \text{Class}(y, c, t_2) \wedge \text{Size}(x, s_1, t_1) \wedge \text{Size}(y, s_2, t_2)] \wedge (s_2 > s_1) \rightarrow (rp_2 > rp_1)).$$

THEOREM 10. *Larger reorganization-free organizations of the same class have higher reliability and accountability*:

$$\forall c, ra_1, ra_2, s_1, s_2, t_1, t_2, x, y([\text{Class}(x, c, t_1) \wedge \text{Class}(y, c, t_2) \wedge \text{Size}(x, s_1, t_1) \wedge \text{Size}(y, s_2, t_2) \wedge \text{Reorg\_free}(x, t_1, t_1) \wedge \text{Reorg\_free}(y, t_2, t_2) \wedge O(x, t_1) \wedge O(y, t_2) \wedge \text{Relacc}(x, ra_1, t_1) \wedge \text{Relacc}(y, ra_2, t_2)] \wedge (s_2 > s_1) \rightarrow (ra_2 > ra_1)).$$

Theorem 11, however, is unexpected: the normal expectation is that organizations can decrease in size without reorganizing. Theorem 11 points either to a weakness in the premise set as the formal representation of OE, or to a limitation of OE itself. On closer inspection, the premise set appears to provide an adequate representation of the theory's assumptions (fortunately, the original text, Hannan and Freeman 1984, gives an explicit list of both assumptions and theorems). So we may conclude that Theorem 11 points to a limitation of OE. OE appears less general than expected, or, to put it more formally, its apparent class of models is smaller than expected. In fact, OE appears to imply a dichotomy between (1) organizations under "normal" conditions and (2) organizations under reorganization. OE's theoretical setup dictates that any decrease in size requires reorganization, so Theorem 11 gives a different meaning to the term "reorganization," or, rather points out how general the meaning of this term is in the theory of OE.

THEOREM 11. *The size of reorganization-free organizations of the same class does not decrease with time*:

$$\forall c, s_1, s_2, t_1, t_2, x([\text{Time}(t_1) \wedge \text{Time}(t_2) \wedge \text{Reorg\_free}(x, t_1, t_2) \wedge \text{Reorg\_free}(x, t_2, t_2) \wedge \text{Reorg\_free}(x, t_1, t_1) \wedge O(x, t_2) \wedge O(x, t_1) \wedge \text{Class}(x, c, t_2) \wedge \text{Class}(x, c, t_1) \wedge \text{Size}(x, s_2, t_2) \wedge \text{Size}(x, s_1, t_1)] \wedge (t_2 > t_1) \rightarrow \neg(s_1 > s_2)).$$

Theorems 12 through 13 are expected, but they, too, show in subtle ways the limits of OE by demonstrating the equivalence of inertia, reliability, and reproducibility. This equivalence is not intended by the original text, but is required to establish the soundness of Theorem 1 (as argued in Péli et al. 1994). By implication, Theorems 12–13 reiterate a problem in the explanatory structure of the original theory.

THEOREM 12. *Organizations with higher reproducibility have higher survival chance*:

$$\forall p_1, p_2, rp_1, rp_2, t_1, t_2, x, y([\text{Repr}(x, rp_1, t_1) \wedge \text{Repr}(y, rp_2, t_2) \wedge O(x, t_1) \wedge O(y, t_2) \wedge \text{Sc}(x, p_1, t_1) \wedge \text{Sc}(y, p_2, t_2)] \wedge (rp_2 > rp_1) \rightarrow (p_2 > p_1)).$$

THEOREM 13a. *Reorganization-free organizations with higher inertia have higher reliability and accountability*:

$$\forall i_1, i_2, ra_1, ra_2, t_1, t_2, x, y([\text{Reorg\_free}(x, t_1, t_1) \wedge \text{Reorg\_free}(y, t_2, t_2) \wedge \text{Iner}(x, i_1, t_1) \wedge \text{Iner}(y, i_2, t_2) \wedge O(x, t_1) \wedge O(y, t_2) \wedge \text{Relacc}(x, ra_1, t_1) \wedge \text{Relacc}(y, ra_2, t_2)] \wedge (i_2 > i_1) \rightarrow (ra_2 > ra_1)).$$

THEOREM 13b. *Reorganization-free organizations with higher reliability and accountability have higher inertia*:

$$\forall i_1, i_2, ra_1, ra_2, t_1, t_2, x, y([\text{Reorg\_free}(x, t_1, t_1) \wedge \text{Reorg\_free}(y, t_2, t_2) \wedge \text{Iner}(x, i_1, t_1) \wedge \text{Iner}(y, i_2, t_2) \wedge O(x, t_1) \wedge O(y, t_2) \wedge \text{Relacc}(x, ra_1, t_1) \wedge \text{Relacc}(y, ra_2, t_2)] \wedge (ra_2 > ra_1) \rightarrow (i_2 > i_1)).$$

The next three theorems, Theorems 14–16, point out some implications of Theorem 5, and so does Theorem 17, but Theorem 17 unexpectedly points to a specific relationship between complexity and size under conditions of reorganization.

THEOREM 14. *More complex organizations of the same class have lower or equal reliability and accountability after reorganizations of the same type*:

$$\forall c, c_1, c_2, p, ra_1, ra_2, re, t_a, t_b, t_c, x, y([O(x, t_a) \wedge O(y, t_a) \wedge \text{Class}(x, c, t_a) \wedge \text{Class}(y, c, t_a) \wedge \text{Sc}(x, p, t_a) \wedge \text{Sc}(y, p, t_a) \wedge \text{Reorg}(x, t_a, t_b) \wedge \text{Reorg}(y, t_a, t_c) \wedge \text{Reorg\_type}(x, re, t_a) \wedge \text{Reorg\_type}(y, re, t_a) \wedge \text{Reorg\_free}(x, t_b, t_c) \wedge \text{Compl}(x, c_1, t_a) \wedge \text{Compl}(y, c_2, t_a) \wedge O(x, t_c) \wedge O(y, t_c) \wedge \text{Relacc}(x, ra_1, t_c) \wedge \text{Relacc}(y, ra_2, t_c)] \wedge (c_2 > c_1) \rightarrow \neg(ra_2 > ra_1)).$$

THEOREM 15. *More complex organizations of the same class have lower or equal reproducibility after reorganizations of the same type*:

$$\forall c, c_1, c_2, p, re, rp_1, rp_2, t_a, t_b, t_c, x, y([O(x, t_a) \wedge O(y, t_a) \wedge \text{Class}(x, c, t_a) \wedge \text{Class}(y, c, t_a) \wedge \text{Sc}(x, p, t_a) \wedge \text{Sc}(y, p, t_a) \wedge \text{Reorg}(x, t_a, t_b) \wedge \text{Reorg}(y, t_a, t_c) \wedge \text{Reorg\_type}(x, re, t_a) \wedge \text{Reorg\_type}(y, re, t_a) \wedge \text{Reorg\_free}(x, t_b, t_c) \wedge \text{Compl}(x, c_1, t_a) \wedge \text{Compl}(y, c_2, t_a) \wedge O(x, t_c) \wedge O(y, t_c) \wedge \text{Repr}(x, rp_1, t_c) \wedge \text{Repr}(y, rp_2, t_c)] \wedge (c_2 > c_1) \rightarrow \neg(rp_2 > rp_1)).$$

THEOREM 16. *More complex organizations of the same class have lower or equal inertia after reorganizations of the same type*:

$$\forall c, c_1, c_2, i_1, i_2, p, re, t_a, t_b, t_c, x, y([O(x, t_a) \wedge O(y, t_a) \wedge \text{Class}(x, c, t_a) \wedge \text{Class}(y, c, t_a) \wedge \text{Sc}(x, p, t_a) \wedge \text{Sc}(y, p, t_a) \wedge \text{Reorg}(x, t_a, t_b) \wedge \text{Reorg}(y, t_a, t_c) \wedge \text{Reorg\_type}(x, re, t_a) \wedge \text{Reorg\_type}(y, re, t_a) \wedge \text{Reorg\_free}(x, t_b, t_c) \wedge \text{Compl}(x, c_1, t_a) \wedge \text{Compl}(y, c_2, t_a) \wedge O(x, t_c) \wedge O(y, t_c) \wedge \text{Reorg\_free}(x, t_c, t_c) \wedge \text{Reorg\_free}(y, t_c, t_c) \wedge \text{Iner}(x, i_1, t_c) \wedge \text{Iner}(y, i_2, t_c)] \wedge (c_2 > c_1) \rightarrow \neg(i_2 > i_1)).$$

THEOREM 17. *More complex organizations of the same class have lower or equal size after reorganizations of the same type*:

$$\forall c, c_1, c_2, new\_c, p, re, s_1, s_2, t_a, t_b, t_c, x, y([O(x, t_a) \wedge O(y, t_a) \wedge \text{Class}(x, c, t_a) \wedge \text{Class}(y, c, t_a) \wedge \text{Sc}(x, p, t_a) \wedge \text{Sc}(y, p, t_a) \wedge \text{Reorg}(x, t_a, t_b) \wedge \text{Reorg}(y, t_a, t_c) \wedge \text{Reorg\_type}(x, re, t_a) \wedge \text{Reorg\_type}(y, re, t_a) \wedge \text{Reorg\_free}(x, t_b, t_c) \wedge \text{Compl}(x, c_1, t_a) \wedge \text{Compl}(y, c_2, t_a) \wedge \text{Reorg\_free}(x, t_c, t_c) \wedge \text{Reorg\_free}(y, t_c, t_c) \wedge O(x, t_c) \wedge O(y, t_c) \wedge \text{Class}(x, new\_c, t_c) \wedge \text{Class}(y, new\_c, t_c) \wedge \text{Size}(x, s_1, t_c) \wedge \text{Size}(y, s_2, t_c)] \wedge (c_2 > c_1) \rightarrow \neg(s_2 > s_1)).$$

In sum, the partial closure through PDC-1 has improved the theory. Theorems 6 through 10 confirm the theoretical expectations. Theorem 11 is unexpected, and contradicts the theoretical expectation. Since the intermediate theory appeared to be adequate, we revised the theoretical expectations instead. The consequence is that the apparent class of models of the theory is reduced. Theorems 12–13 are expected (given the revised theoretical expectations). The next three theorems, Theorems 14–16, strengthen the original theoretical expectations, but the last, Theorem 17, is also unexpected and forces an update of the theoretical expectations.

The premises of the inertia fragment and the theorems derived by PDC-1 are shown in Figure 2. The nodes represent the theorems (a theorem relates the top-node with the node); the arrows denote the premises that constitute the theorems.

7. Discussion and Conclusions

We have argued that formal logic helps researchers to improve a theory in various ways. In a static perspective, logic can help them to answer questions of consistency and explanatory soundness. In a dynamic perspective, logic can help to discover hidden implications of a given theory, or, more precisely, implications of a formal representation of the theory in logical terms. As the logical cycle demonstrates, the partial closure can advance a theory in various ways, either by reinforcing original theoretical expectations about a domain, or by suggesting a modification of those expectations. Conversely, if there is no reason to modify the expectations, the partial closure can point out weaknesses in the formal representation of the theory.

PDC-1 has its limits, of course.

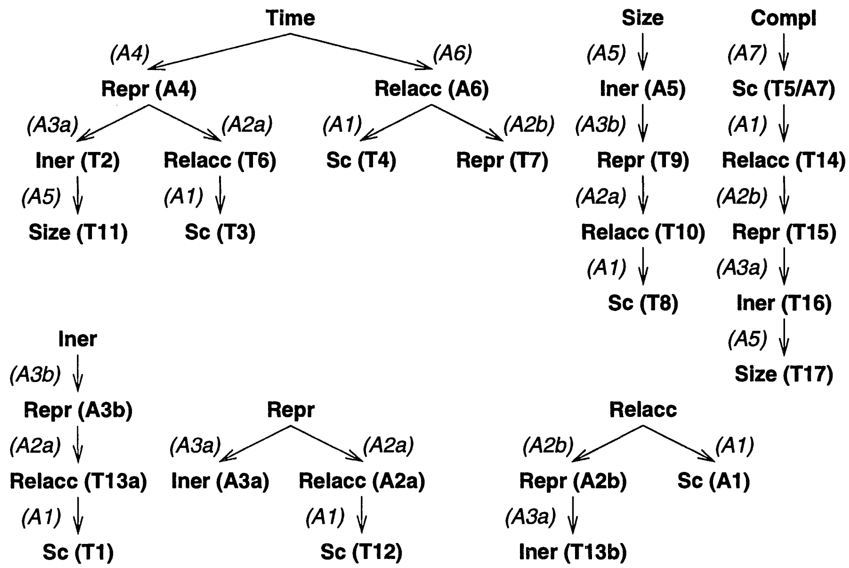
First, the algorithm works only on a fragment of FOL, namely on formulas that we called "single property to single property." This fragment is arguably important—important enough to allow for a formalization of the inertia part of OE—but it does not have the full expressive power of FOL.

Second, FOL itself has its limits. There has been a lively debate in philosophy about the use of FOL as a tool for formalizing scientific theories (Ayer 1959). Many scientific theories involve notions that FOL cannot handle directly, such as counterfactual conditions and intensional constructs. One could even argue that scientific laws are not material implications as provided by FOL. Because of this, there is a broad agreement that FOL on its own is too weak to formalize all scientific theories. However, our goals are more modest; we focus on theories that do not transcend the expressive boundaries of FOL. Even if scientific laws are not material implications they do imply material implications (as was kindly pointed out to us by one reviewer), and it remains of interest to generate implicit consequences thereof. *Organizational ecology*, the domain theory examined in this article, is restricted to object-related statements about properties and relations. But *action theories*, an important class of theories in the social sciences, do involve opaque contexts that are created by actions; for such theories, modal action logics appear more appropriate. Evidence on the formalization of J. D. Thompson's *Organization in Action* (Masuch and Huang 1996) shows, for example, that extending FOL with modal (intensional) operators facilitates the formal representation of an action theory considerably.

Third, FOL is not necessarily the most elegant or efficient language. Once a representation in FOL has been generated for a specific domain, simpler, or more parsimonious representations may suggest themselves (Newell and Simon 1972, Brachman and Levesque 1985). But the general experience in natural language representation points to a trade off between specificity and flexibility. For specific domains, specialized languages may appear more appropriate, but such languages are not easily generalizable to other domains. Conversely, a general language may not give the most efficient representation for a particular domain, but it carries over more easily to other domains; because of this, a general language is more appropriate for a generic application for theorem-finding. Using FOL has one additional, very important advantage: its formal properties are well-understood. The formal properties of specialized ad hoc languages are, as a rule, not known. For example, without a formal semantics, one has no criteria for soundness; without a proof theory, one has no machinery for derivations.

Fourth, PDC-1 is restricted to finding the logical implications of a given set of premises. It cannot generate new conclusions in a logical sense. In fact, once the logic plus a set of premises are fixed, no deductive procedure

Figure 2 The Assumptions (A*n*) and Theorems (T*n*) of the Inertia Fragment



can generate logically new conclusions; logically new conclusions require new premises or a new logic. The motivation for this research was to generate theorems that are "new" in an *empirical* sense: implied by the logic but (perhaps) unknown to the researcher. Such conclusions may or may not be of particular interest—in this sense the choice to focus on SPtSP formulas is of a heuristic nature. There is no guarantee that PDC-1 (or more general algorithms, for that matter) will always generate interesting (empirically) new theorems. In our case study the algorithm does generate interesting theorems, but more cases are needed to settle this empirical matter.

The usefulness of PDC-1 was demonstrated for an important organization theory, Organizational Ecology; the algorithm generated a set of new theorems, including some of real theoretical importance, notably Theorem 11. The fact that the theory implies that organizations cannot decrease in size under normal conditions is clearly important for gauging the OE's scope and setup. Of course, the algorithm's job could also have been done "by hand." In fact, some of the new theorems had already been detected by hand, as reported in Péli et al. (1994); but then, some had not.

Our tool can also be used during the original formalization of the theory. Recall that the algorithm also identified all previously known theorems from the premises, so it could have been used to check the soundness of the original theory. In fact, the investigation of the inertia fragment by our method would have revealed several more or less serious flaws in the explanatory structure of the original presentation of the theory. In particular, it would have pointed out that the derivation of Theorem 1 (selection favors inertia), arguably the most important theorem of the inertia part, is unsound, and that a sound derivation requires additional qualifications that reduce the scope of the original theory quite considerably; see Péli et al. (1994). Furthermore, the algorithm can help to enlarge the scope of the theory by helping the theoretician to find out "what would happen if" he would add new assumptions to the original premise set. In this way, PDC-1 makes an important step towards an application for logical simulation.

As a direct follow-up of the reported research, we want to extend the PDC-1 algorithm to other classes of theorems. Taking into account that a complete deductive closure will comprise infinitely many theorems (most of them completely uninteresting), extending the PDC-1 algorithm should be done with care. The job of the algorithm is not only to derive a particular class of theorems, but also to ensure that nothing else is derived.[2]

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