On the Process of Axiomatizing Scientific Theories: Using Justification Criteria in the Context of Discovery

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Abstract

This paper reports on a concerted effort to axiomatize social science theories in first order logic. Such an axiomatization is traditionally viewed as the ultimate step in the justification of a theory. A first order logic rendition of a theory gives an explicit, unambiguous exposition of the theory. This allows, in turn, for testing for a number of criteria (such as consistency, soundness of derivations, satisfiability and falsifiability of theorems) that can be evaluated using generic tools from automated reasoning. We can make a rigorous evaluation of a scientific theory by assessing these criteria. However, as it turns out, these criteria exceed their use as rigid, final tests and are especially useful during the process of formalizing a theory. The criteria can provide useful feedback on how to revise the theory in case of a deficiency, for example, they can identify implicit (background) assumptions of the theory. As a result, the tools of the context of justification can also play an important role in the context of discovery.

1 Introduction

This paper reports on a concerted effort to axiomatize social science theories in first order logic. In recent years, several authors started working on the formal reconstruction of ordinary language, social science theories. The main focus is on organization theory, and especially on the branch based on a natural selection perspective on organizations (Hannan and Freeman, 1989). This resulted in the logical formalization of axiomatization of several parts of "organizational ecology": The inertia theory was formalized in (Péli et al., 1994); the life-history theory in (Péli and Masuch, 1997); the niche width theory in (Péli, 1997; Bruggeman, 1997); and the age-dependence theory in (Hannan, 1998). Further efforts were directed at a classic version of contingency theory, Thompson (1967) "Organizations in Action," as formalized in (Kamps and Pólos, 1999) and at several relatively simple theories, such as Hage (1965) "An axiomatic theory of organizations" as formalized in (Kamps, 1998) and a theory from Zetterberg (1965) "On Theory and Verification in Sociology" that we treat as a case study in this paper. These authors presented first order logic versions of previously non-formal scientific theories, by reconstructing the textual argumentation, and rendering this in formal logic. A notable exception is Hannan (1998), who uses the logical formalization to develop new theory "to clarify an area of research characterized by conflicting claims and divergent empirical findings" (p.126).

Following Reichenbach (1938), scientific activities are traditionally dichotomized into the context of justification and the context of discovery. The axiomatization of theories in (first-order) logic is traditionally viewed as the final step in the justification of a theory. A first order logic rendition of a theory gives an explicit, unambiguous exposition of the theory. This allows, in turn, for testing for a number of criteria (such as consistency, soundness of derivations, satisfiability and falsifiability of theorems) that can be evaluated using generic tools from automated reasoning. We can make a rigorous evaluation of a scientific theory by assessing these criteria. However, as it turns out, these criteria exceed their use as rigid, final tests and are especially useful during the process of formalizing a theory. The criteria can provide useful feedback on how to revise the theory in case of a deficiency, for example, they can identify implicit (background) assumptions of the theory. As a result, the tools of the context of justification can also play an important role in the context of discovery.

This paper is structured as follows: First, in §2, we will discuss the axiomatization of theories; next, in §3, we will illustrate this by axiomatizing the theory of (Zetterberg, 1965); and finally, in §4, we will draw conclusions and discuss issues related to our work.

2 Axiomatizing Theories

2.1 The product of formalization

The axiomatization or logical formalization of non-formal theories consists of the interlinked activities of rational reconstruction (reconstructing the claims, premises, and argumentation of a theory) and formal modeling (capturing the claims as theorems that are provable from explicit assumptions). The main benefit of the formalization of theories in logic is that it provides clarity (Suppes, 1968). It will provide a unambiguous exposition of the theory, containing explicit axioms and theorems. Moreover, the logic allows us to formulate a number of criteria for evaluating theories. These criteria can be used for assessing relevant properties of the theory.

2.1.1 Criteria for evaluating theories

We use the following criteria for evaluating theories (Kamps, 1998, 1999):

Consistency The *consistency* of the formal theory ensures that the theory is free of contradictions. Ordinary language theories do rarely contain conspicuous contradictions, since they are easily obscured by the ambiguity of natural language. When formalizing a theory in logic, these contradictions will surface and can be resolved. We can prove the consistency of a theory by generating a model of the premises. Alternatively, we can prove the inconsistency of a theory by deriving a contradiction from (some subset of) the premises.

Soundness The argumentation for a proposition is *sound* if the proposition is a logical consequence of the premises (i.e., if the proposition is a theorem). That is, if we consider cases in which the premises hold, then the theorem must also hold (i.e., the theorem is a prediction). Conversely, if we consider cases where the theorem holds, then the premises give an explanation for why the theorem holds. Many of our basic propositions are inaccessible for direct empirical testing. Such propositions can be indirectly tested by their testable implications (Hempel, 1966). The soundness criterion allows us to identify implications of inaccessible propositions that can be subjected to empirical testing. We can establish the soundness of a derivation by proving a theorem from the premises. We can prove that the derivation is unsound by constructing a model of the premises in which the theorem is false.

Falsifiability Falsifiability is an essential property of scientific theories (Popper, 1959). If no state of affairs can falsify a theory, empirical testing can only reassert its trivial validity. If we can construct a model in which the theorem is false (including further only the definitions in the premise set), then we have proven that a theorem is falsifiable. If we can prove a theorem from an empty premise set (or from just the definitions in the premise set), we have shown that the theorem is unfalsifiable or self-contained—it's truth does not depend on the other premises of the theory.

Satisfiability Satisfiability of a theorem ensures that it can be fulfilled. If we can construct a model in which the theorem is holds (including only the definitions in the premise set), then we have proven that a theorem is satisfiable. If we can derive a contradiction of the theorem (plus the definitions in the premise set), we have shown that the theorem is unsatisfiable or self-contradictory. Theorems that are both satisfiable and falsifiable are *contingent*—their validity strictly depends on the premises, they are neither tautologically true, nor self-contradictory.

Further advantages Making the inference structure of a theory explicit make it also possible to assess its *parsimony* (for example, it may turn out that some assumption are not necessary, or can be relaxed), *explanatory and predictive power* (by looking at the theorems, because these are the predictions of the theory, and the proofs give explanations for them), *coherence* (for example, a theory may turn out to have unrelated or independent parts), and other such properties. We can investigate the *domain* or scope of a theory by investigating the models of the theory.

2.1.2 Computational support

The axiomatization of any substantive domain in first order logic typically requires various tedious calculations. Generic tools from the domain of automated reasoning can be used for computational testing of the theoretical criteria (Kamps, 1998). We use both automated theorem provers and model generators. Automated theorem provers, such as OTTER (McCune, 1994b), typically attempt to prove theorems by *reductio ad absurdum*, that is, the program attempts to derive a contradiction from the premises and the negation of the theorem. Theorem provers can also be used to prove that a theory is inconsistent by deriving a contradiction from only the set of premises. Automated model generators, such as MACE (McCune, 1994a), can find (small) models of sets of sentences. A model generator can prove the consistency of a theory, if it can generate a model of the premises. They can also be used to prove underivability of a conjecture, if it can generate a model of the premises in which the conjecture is false. The specific queries for the criteria are summarized in Table 1.

Table 1: The criteria and automated reasoning tools.

CRITERION	OTTER	MACE
Consistency		$(\exists \mathcal{A})\ \mathcal{A} \models \Sigma$
Inconsistency	$\Sigma \vdash \perp$	
Soundness	$\Sigma \cup \{\neg\varphi\} \vdash \perp$	
Unsoundness		$(\exists \mathcal{A})\ \mathcal{A} \models \Sigma \cup \{\neg\varphi\}$
Falsifiability		$(\exists \mathcal{A})\ \mathcal{A} \models \Sigma_{\mathsf{def}} \cup \{\neg\varphi\}$
Unfalsifiability	$\Sigma_{\mathsf{def}} \cup \{\neg\varphi\} \vdash \perp$	
Satisfiability		$(\exists \mathcal{A})\ \mathcal{A} \models \Sigma_{\mathsf{def}} \cup \{\varphi\}$
Unsatisfiability	$\Sigma_{\mathsf{def}} \cup \{\varphi\} \vdash \perp$	

Note: Σ denotes a premise set, with Σ_{def} the definitions in this set, and φ a conjecture or theorem.

2.2 The process of formalization

Most social science theories are stated in ordinary language (for example in articles in social science journals). As a result, the main obstacle for logical formalizing such a discursive theory is their rational reconstruction: interpreting the text, distinguishing important claims and argumentation from other parts of the text, and reconstructing the argumentation. This reconstruction is seldom a straightforward process, although there are some useful guidelines (see, for example, the method in Fisher, 1988). When the theoretical statements are singled-out, they can be formulated in first order logic. This initial formalization can be evaluated by the criteria of §2.1.1, such as consistency, and soundness of arguments.

A strict justification point of view would stop after evaluating the theory by these criteria. However, since theories stated in ordinary language are typically partial and incomplete, it is highly unlikely that our initial formalization of the theory is completely satisfactory. For example, the initial formalization may turn out to be inconsistent, or some of the claims may not be derivable. Finding such undesirable properties does reveal deficiencies of our initial formal rendition of the theory. Is it justified to pass this verdict on to the original theory? It seems not, and as a result, we attempt to revise our initial formalization such that it is a better reconstruction of the original theory. Fortunately, analyzing the criteria can provide useful feedback for the revision of our initial formalization.

2.2.1 Recover from inconsistency

Although theories in natural language rarely contain conspicuous contradictions, the ambiguities of ordinary language can easily obscure them. As a result, our initial formalization of a theory may easily turn out to be inconsistent.

The proof of the inconsistency of the theory (as provided by an automated theorem prover) is a derivation of a contradiction from a specific subset of the premises. Examining this proof will clarify what caused the inconsistency, which may, in turn, suggest how to resolve the contradiction. For example, by changing a definition, or by making some assumption weaker. After revising the premises we have to repeat the test for consistency, to ensure that the modifications are sufficient. Also, a premise set may contain several different contradiction. In these cases, repeated testing for inconsistencies allows for a piecemeal revision of the theory.

2.2.2 Recover from unsound argumentation

Since authors typically assume a body of common background knowledge, it is unlikely that all needed premises are mentioned in the source text. One of the biggest problems during the logical formalization of a theory, is to make these implicit background assumptions explicit. As a result, some of the theory's claim may turn out to be underivable in our initial formalization of the theory. Of course, it can also be the case that the claim turns out to be a false conjecture.

The proof that the derivation of a claim is unsound (as provided by an automated model generator) is a counterexample, that is, a model of the premises in which the claim is false. Examining this model gives important feedback for possible revision of the theory. On the one hand, it may be the case that, although a model of the premises, it is a non-intended model in the sense that we did not intend this model to belong to the theory. For example, the model may conflict with our common sense, or with common background knowledge in the domain of the theory. In this case we have to add this common sense or background knowledge to the premises of the theory, and see if we can now derive the claim. There may be several assumptions missing, which give rise to different counterexample. On the other hand, the model may be a intended model of the theory and in this model the claim is false. In this case, the original claim is too strong, and we have to modify it. A typical example is the case in which the claim is overstated, and the model presents an exception that should be excluded. After revising the claim we can test whether we can derive this weaker claim. There may be other exceptions to the claim and repeated tests for unsoundness allows for a piecemeal treatment of them. In some cases, the claim may have to be retracted altogether, or we are forced to add further assumptions that will restrict the theory's domain of application.

As a result, the formalization of theories proceeds in a cyclic process, in which the formal theory is repeatedly revised. Moreover, these modifications may have an impact on the original theory. Consider the case in which the original theory is inconsistent. If we can resolve the inconsistency in the formal rendition, we can translate this revision back to the original theory. Consider the case in which the original theory contains hiatus. If we can find reasonable assumptions that make the claims derivable in the formal theory, we can, again, translate these added assumptions back to the original theory. It may also be the case that the formal theory reveals that certain restricting assumptions of the original theory are not necessary or can be relaxed. In short, in these cases the formal theory and the original theory evolve in parallel.

3 Case Study

This section contains a logical formalization of Zetterberg's '*On Theory and Verification in Sociology*' (Zetterberg, 1965). Zetterberg (1965) is stated in ordinary language—it is not a formal theory—but the main propositions are clearly outlined. It is an 'axiomatic theory' consisting of 10 propositions. The ten propositions are about five 'variables' of social groups (p.159): 1) the number of associates per member in the group; 2) the solidarity of the group; 3) the consensus of the beliefs, values, and norms in the group; 4) the division of labor in the group; and 5) the extent to which persons are rejected (excluded) from the group when they violate norms.

Table 2 lists ten propositions relating these five variables. Zetterberg regards the last four propositions (labeled 7 through 10) as axioms of the theory. These last four propositions bear some relation to Durkheim's classical work on the division of labor (Durkheim, 1893). The first six propositions (labeled 1 through 6) are claimed to

Table 2: The propositions of Zetterberg's theory.

P.1	*The greater the division of labor, the greater the consensus.*
P.2	*The greater the solidarity, the greater the number of associates per member.*
P.3	*The greater the number of associates per member, the greater the consensus.*
P.4	*The greater the consensus, the smaller the number of rejections per deviant.*
P.5	*The greater the division of labor, the smaller the number of rejections per deviant.*
P.6	*The greater the number of associates per member, the smaller the number of rejections per deviant.*
P.7	*The greater the division of labor, the greater the solidarity.*
P.8	*The greater the solidarity, the greater the consensus.*
P.9	*The greater the number of associates per member, the greater the division of labor.*
P.10	*The greater the solidarity, the smaller the number of rejections per deviant.*

Note: All propositions are from Zetterberg (1965, pp.159–160).

be derivable from the four axioms using "the deduction rules of ordinary language" (p.163). According to Zetterberg, we can make the following derivations (p.161): Proposition **P.1** can be inferred from **P.7** and **P.8**; Proposition **P.2** from **P.7** and **P.9**; Proposition **P.3** from **P.8** and derived proposition **P.2**; Proposition **P.4** from **P.8** and **P.10**; Proposition **P.5** from **P.7** and **P.10**; and Proposition **P.6** from **P.9** and derived proposition **P.5**.

Only few social science theories contain an explicit inference structure between their propositions (except, of course, for mathematical theories in economics). Although not a prototypical sociological theory, its axiomatic structure makes Zetterberg (1965) a very suitable candidate for a case study. We can focus on the formal modeling of the theory without doing an extensive reconstruction of the argumentation.

3.1 Formalization

3.1.1 Functions and predicates

We use first order logic to formalize Zetterberg's propositions. In the formalization we use unary functions to represent the five variables (see Table 3).

Table 3: Functions and predicates.

$\mathsf{napm}(x)$	number of associates per member in group x
$\mathsf{soli}(x)$	solidarity of group x
$\mathsf{cons}(x)$	consensus of the beliefs, values, and norms in group x
$\mathsf{dlab}(x)$	division of labor in group x
$\mathsf{nrpd}(x)$	number of rejections per deviant in group x
$x > y$	x is greater than y (or y is smaller than x)

3.1.2 Axioms

Zetterberg (1965, p.160) regards Propositions 7 through 10 as axioms of the theory. A rendering of these propositions in first order logic is presented in Table 4.

Table 4: A formalization of Propositions 7–10.

A.7	$\forall x, y\ [\mathsf{dlab}(x) > \mathsf{dlab}(y) \ \rightarrow\ \mathsf{soli}(x) > \mathsf{soli}(y)]$
A.8	$\forall x, y\ [\mathsf{soli}(x) > \mathsf{soli}(y) \ \rightarrow\ \mathsf{cons}(x) > \mathsf{cons}(y)]$
A.9	$\forall x, y\ [\mathsf{napm}(x) > \mathsf{napm}(y) \ \rightarrow\ \mathsf{dlab}(x) > \mathsf{dlab}(y)]$
A.10	$\forall x, y\ [\mathsf{soli}(x) > \mathsf{soli}(y) \ \rightarrow\ \mathsf{nrpd}(y) > \mathsf{nrpd}(x)]$

Is the (formal) theory consistent? The theory is consistent if it has a model. We used the automated model generator MACE in an attempt to generate models of the axioms. Table 5 shows a model of Propositions 7–10.[1] We can easily verify that the axioms hold in the model of Table 5: Axiom 7 holds because the group with greater division of labor (0) has also a greater solidarity; Axiom 8 holds because the group with greater solidarity (0) also has greater consensus; Axiom 9 holds vacuously because there is no group with greater number of associates per member; and finally, Axiom 10 holds because the group with greater solidarity (0) has a smaller number of rejections per deviant. Zetterberg's theory has a model, therefore it is consistent.

Table 5: A model of Axioms 7–10.

	napm	soli	cons	dlab	nrpd
0	0	1	1	1	0
1	0	0	0	0	1

$>$	0	1
0	F	F
1	T	F

3.1.3 Theorems

We will now investigate the explanatory or predictive power of the theory. Zetterberg claims that "these four propositions [7–10] can be used to derive the other findings which thus become theorems" (p.161). In these derivation he uses "the deduction rules of natural language" (p.163). A first order logic rendering of these (intended) theorems is presented in Table 6.

Table 6: A formalization of Propositions 1–6.

T.1	$\forall x, y\ [\mathsf{dlab}(x) > \mathsf{dlab}(y) \ \rightarrow\ \mathsf{cons}(x) > \mathsf{cons}(y)]$
T.2	$\forall x, y\ [\mathsf{soli}(x) > \mathsf{soli}(y) \ \rightarrow\ \mathsf{napm}(x) > \mathsf{napm}(y)]$
T.3	$\forall x, y\ [\mathsf{napm}(x) > \mathsf{napm}(y) \ \rightarrow\ \mathsf{cons}(x) > \mathsf{cons}(y)]$
T.4	$\forall x, y\ [\mathsf{cons}(x) > \mathsf{cons}(y) \ \rightarrow\ \mathsf{nrpd}(y) > \mathsf{nrpd}(x)]$
T.5	$\forall x, y\ [\mathsf{dlab}(x) > \mathsf{dlab}(y) \ \rightarrow\ \mathsf{nrpd}(y) > \mathsf{nrpd}(x)]$
T.6	$\forall x, y\ [\mathsf{napm}(x) > \mathsf{napm}(y) \rightarrow \mathsf{nrpd}(y) > \mathsf{nrpd}(x)]$

[1] On domain size 2, MACE generates 1024 models of the axioms.

We used the automated theorem prover OTTER to attempt proving Propositions 1–6 from the four axioms. We can find proofs of Proposition 1, 3, 5, and 6.

Theorem 1 *The greater the division of labor, the greater the consensus*:

$$\forall x, y\ [\mathsf{dlab}(x) > \mathsf{dlab}(y) \ \rightarrow\ \mathsf{cons}(x) > \mathsf{cons}(y)]$$

(Proof: OTTER can derive **T.1** from **A.7** and **A.8**.)

Theorem 3 *The greater the number of associates per member, the greater the consensus*:

$$\forall x, y\ [\mathsf{napm}(x) > \mathsf{napm}(y) \ \rightarrow\ \mathsf{cons}(x) > \mathsf{cons}(y)]$$

(Proof: OTTER can derive **T.3** from **A.9**, **A.7**, and **A.8**.)

Theorem 5 *The greater the division of labor, the smaller the number of rejections per deviant*:

$$\forall x, y\ [\mathsf{dlab}(x) > \mathsf{dlab}(y) \ \rightarrow\ \mathsf{nrpd}(y) > \mathsf{nrpd}(x)]$$

(Proof: OTTER can derive **T.5** from **A.7** and **A.10**.)

Theorem 6 *The greater the number of associates per member, the smaller the number of rejections per deviant*:

$$\forall x, y\ [\mathsf{napm}(x) > \mathsf{napm}(y) \ \rightarrow\ \mathsf{nrpd}(y) > \mathsf{nrpd}(x)]$$

(Proof: OTTER can derive **T.6** from **A.9**, **A.7**, and **A.10**.)

However, we cannot derive Propositions 2, the greater the solidarity, the greater the number of associates per member:

$$\forall x, y\ [\mathsf{soli}(x) > \mathsf{soli}(y) \ \rightarrow\ \mathsf{napm}(x) > \mathsf{napm}(y)]$$

Nor can we derive Proposition 4, the greater the consensus, the smaller the number of rejections per deviant:

$$\forall x, y\ [\mathsf{cons}(x) > \mathsf{cons}(y) \ \rightarrow\ \mathsf{nrpd}(y) > \mathsf{nrpd}(x)]$$

According to Zetterberg is Proposition 2 derivable from Proposition 7 and 9 (p.161). We can prove that the derivation of **T.2** is unsound if we can find a counterexample, that is, if we can find a model of the axioms in which the intended theorem does not hold. As it turns out, we can construct a counterexample to this claim (see Table 7).

Table 7: Counterexample to Propositions 2.

	napm	soli	cons	dlab	nrpd
0	0	0	0	0	0
1	1	0	0	0	0

>	0	1
0	T	F
1	F	F

Finding this counterexample proves that **T.2** is not a consequence of **A.7–10**.[2] Zetterberg also claims that Proposition 4 is derivable from Proposition 8 and 10 (p.161). Again, we can construct a counterexample to this claim (see Table 8). These counterexamples prove that Propositions 2 and 4 are no theorems of the axioms. On the positive side, we can derive a new proposition as theorem:

Table 8: Counterexample to Propositions 4.

	napm	soli	cons	dlab	nrpd
0	0	0	0	0	0
1	0	0	1	0	0

>	0	1
0	F	T
1	F	F

[2] We are able to prove theorem **T.3** despite the suggestion that its proof depends upon theorem **T.2**. Fortunately, we could find a (different) proof of **T.3** using axioms **A.9**, **A.7**, and **A.10** (or, equivalently, using axiom **A.9** and theorem **T.1**). In fact, the situation would not change much if we could prove **T.2**, since the derivation of **T.3** from **T.2** and **A.8** is unsound!

Theorem 11 *The greater the number of associates per member, the greater the solidarity*:

$$\forall x, y\ [\mathsf{napm}(x) > \mathsf{napm}(y) \ \rightarrow\ \mathsf{soli}(x) > \mathsf{soli}(y)]$$

(Proof: OTTER can derive **T.11** from **A.9** and **A.7**.)

Theorem 11 is the converse of the (underivable) Proposition 2. Of course, there are also many trivial theorems that can be derived, such as all tautologies or theorems already subsumed by the spelled-out theorems.

We formalized Zetterberg's propositions as **T.1–6** and **A.7–10** in Table 6 and 4 respectively. The resulting theory contains propositions **T.1**, **T.3**, **T.5**, and **T.6** as theorems that can be derived from the four axioms, **A.7–10**. The proofs of **T.1**, **T.5**, and **T.6** are according to the suggested inferences. However, we proved that propositions **T.2** and **T.4** cannot be derived and are no theorems of the theory. Although the proof of **T.3** is suggested to depend on the underivable proposition **T.2**, we can find a different proof of **T.3**.

3.2 Revision

Our formalization of Zetterberg's theory prompts a number of questions: How do these results in the formal version of the theory relate to the original version? Have we uncovered a deficiency in the original theory? Went something wrong in our reconstruction of his arguments? Can we come up with a different interpretation in which the intended theorems are derivable? We will try to answer these questions in this section.

3.2.1 Limit explanatory/predictive power

One option is to do nothing: We have given, arguably, the most natural first order rendition of the propositions. If two of the conjectures are not derivable in the formal version of the theory, then we have an important argument to discard these propositions as false conjectures. That is, we keep Axioms 7–10 as stated formalized in Table 4, and reduce the set of theorems to Theorem 1, 3, and 5–6 as stated in Table 6. As Zetterberg (1965, p.163) remarks, "our deductions are not too precise, so long as our concepts are defined in normal prose, and the deduction rules of ordinary language are used." It may be not unreasonable to assume that now that we use formal logic, having a precisely defined language and notion of deduction, we have to discard two of the intended, ordinary language theorems as false conjectures.

3.2.2 Nonintended models and real counterexamples

Before passing such a severe verdict on the theory, it seems more reasonable to first make a detailed examination of the evidence. It is important to note that we did more than proving that the two conjectures are underivable, since we produced the counterexamples that prove the underivability. These counterexamples are available for inspection.

When analyzing at the counterexample of Table 7, we immediate find a strange feature. The model gives a unnatural interpretation of the "$>$"-relation: $(0 > 0)$ is true, whereas $(0 > 1)$, $(1 > 0)$, and $(1 > 1)$ are false. This model is not one of the models we intended to be models of the theory. It seems unreasonable to discard a conjecture because of the existence of such an unintended model. Any exposition of a theory presupposes a common set of background knowledge. In a formal exposition of a theory, relevant parts of this implicit background knowledge have to be added explicitly to the theory. We would assume that the "$>$"-relation denotes a strict larger relation (Meaning Postulate 1) and that on the domain $\{0, 1\}$ it holds that $(1 > 0)$ (Meaning Postulate 2). We decide to add these axioms to the theory (see Table 9).

Table 9: Background assumptions.

MP.1	$\forall x, y\, \neg[(x > y) \wedge (y > x)]$
MP.2	$(1 > 0)$.

Adding these two background assumptions to the theory will substantially reduce the number of models of the theory. Specifically, and more importantly, the models in Tables 7 and 8 (that were counterexamples to **T.2** and **T.4**) do no longer belong to the theory. This mean that we can retry to prove **T.2** and **T.4**. Unfortunately, OTTER is still unable to prove either of them: there must exist other counterexamples to Propositions 2 and 4.

MACE proves that Proposition 2 is still underivable by generating the counterexamples in Table 10 (w.l.o.g., we use only **A.7**, **A.9**, **MP.1**, and **MP.2**).[3] Similarly, for Proposition 4 in Table 11 (w.l.o.g., we use only **A.8**, **A.10**, **MP.1**, and **MP.2**).[4] The models in Tables 10 and 11 are genuine counterexamples. We will now explore different ways of dealing with them.

Table 10: Counterexamples to Propositions 2.

	soli	dlab	napm
0	0	0	0
1	1	0	0
0	0	0	1
1	1	0	1
0	0	0	0
1	1	1	0
0	0	0	1
1	1	1	1
0	0	1	0
1	1	1	0
0	0	1	0
1	1	1	0

$>$	0	1
0	F	F
1	T	F

Table 11: Counterexamples to Propositions 4.

	cons	soli	nrpd
0	0	0	0
1	1	0	0
0	0	0	0
1	1	0	1
0	0	0	0
1	1	0	1
0	0	1	0
1	1	1	0
0	0	1	0
1	1	1	1
0	0	1	1
1	1	1	1

$>$	0	1
0	F	F
1	T	F

3.2.3 Weaken theorems

This option means that we do regard the counterexamples as faithful models of the theory. Therefore, if we want retain Propositions 2 and 4, we must reformulate them such that they hold in the models that are counterexamples to their initial formulation, e.g., in models such as in Tables 10 and 11 respectively. In order to hold in a larger set models, the reformulated propositions must be weaker than the original formulations. We will attempt to find reformulated propositions that are provable from the axioms, yet still close to the original formulation of the theorems. Note that this does not change the theory: we will only change our exposition of the theory by singling out more consequences of the axioms explicitly. Since all consequences of the axiom set are, by definition, part of the theory, the theory does not change if we single out more of them.

In case of Proposition 2, '*the greater the solidarity, the greater the number of associates per member*', this means that the new formulation must hold in (at least) the models of Table 10. Let us analyze these models: they all have the form $\mathsf{soli}(1) > \mathsf{soli}(0)$ and $\mathsf{napm}(0) = \mathsf{napm}(1)$. A weaker version of Proposition 2 that also holds in these models is: '*The greater the solidarity, the greater* ***or equal*** *the number of associates per member*':

T.2$^-$ $\forall x, y\ [\mathsf{soli}(x) > \mathsf{soli}(y) \rightarrow \neg(\mathsf{napm}(y) > \mathsf{napm}(x))]$

Now that we have reformulated Proposition 2, we can retry to prove it. (We have not changed the axioms, so the model in Table 5 still proves that the theory is consistent.) Although we have dealt with the (type of) counterexamples in Table 10, there may still be other counterexamples. There turn out to be none, since we can prove the reformulated theorem.

[3] MACE finds twelve models on domain size 2. Table 10 lists six of them, the other six are isomorphic copies with arguments 0 and 1 interchanged.

[4] MACE finds twelve models on domain size 2. Table 11 prints six of them, the other six are isomorphic copies with arguments 0 and 1 interchanged.

Theorem 2$^-$ *The greater the solidarity, the greater or equal the number of associates per member*:

$$\forall x, y\ [\mathsf{soli}(x) > \mathsf{soli}(y)\ \rightarrow\ \neg(\mathsf{napm}(y) > \mathsf{napm}(x))]$$

(Proof: OTTER can derive **T.2$^-$** from **A.7**, **A.9**, and **MP.1**.)[5]

We can try to apply the same strategy to Proposition 4. Let us analyze the models in Table 11. There are counterexamples of the form $\mathsf{cons}(1) > \mathsf{cons}(0)$ and $\mathsf{nrpd}(0) = \mathsf{nrpd}(1)$ and counterexamples of the form $\mathsf{cons}(1) > \mathsf{cons}(0)$ and $\mathsf{nrpd}(0) > \mathsf{nrpd}(1)$. Moreover, there are also models in which Proposition 4 holds (so, these are not counterexamples), and these have the form $\mathsf{cons}(1) > \mathsf{cons}(0)$ and $\mathsf{nrpd}(1) > \mathsf{nrpd}(0)$ (for example, the model in Table 5).

It seems like the axioms put hardly any constraint on the relation between consensus and number of rejections per deviant! A weaker version of Proposition 4 that holds in all these models must be very weak—so weak that is a tautology. For example: '*The greater the consensus, the smaller,* ***or equal, or higher*** *the number of rejections per deviant.*'

We can derive a (weaker) version of Proposition 2 (Theorem 2$^-$), but this does not help in deriving Proposition 4.

3.2.4 Strengthen axioms

A final option is to regard the counterexamples as models that are outside the domain of the theory. That is, the theorems do hold but on a smaller domain. Therefore, we must reformulate the axioms such that models such as those in Tables 10 and 11 are no longer models of the revised axioms. In order to hold in a smaller set of models, the revised axioms must be stronger than the original axioms. There are several ways to make Proposition 2 derivable. We choose a way which follows the original argumentation as closely as possible.

Zetterberg argues that Proposition 2 is derivable from Proposition 7 and 9. As stated above, the counterexamples of Table 10 have the form $\mathsf{soli}(1) > \mathsf{soli}(0)$ and $\mathsf{napm}(0) = \mathsf{napm}(1)$. A natural way to exclude these counterexamples is to add as an axiom that *the greater the solidarity, the greater the number of associates per member*:

$$\forall x, y\ [\mathsf{soli}(x) > \mathsf{soli}(y)\ \rightarrow\ \mathsf{napm}(x) > \mathsf{napm}(y)]$$

However, that would mean that we add Proposition 2 as an axiom (which means that we can trivially derive it).

A second way to exclude the counterexamples is to add as axioms that *the greater the solidarity, the greater the division of labor* (the converse of Proposition 7):

A.12 $\forall x, y\ [\mathsf{soli}(x) > \mathsf{soli}(y)\ \rightarrow\ \mathsf{dlab}(x) > \mathsf{dlab}(y)]$

and *the greater the division of labor, the greater the number of associates per member* (the converse of Proposition 9):

A.13 $\forall x, y\ [\mathsf{dlab}(x) > \mathsf{dlab}(y)\ \rightarrow\ \mathsf{napm}(x) > \mathsf{napm}(y)]$

Adding these two axioms takes care of (the type of) counterexamples in Table 10 (Axiom 12 removes models 1–2, and 5–6; Axiom 13 removes models 3–4). We can now make a new attempt at proving Proposition 2 using the revised set of axioms. If this attempt succeeds, the revision has removed all counterexamples. As it turns out, we can indeed prove Proposition 2.

Theorem 2 *The greater the solidarity, the greater the number of associates per member*:

$$\forall x, y\ [\mathsf{soli}(x) > \mathsf{soli}(y)\ \rightarrow\ \mathsf{napm}(x) > \mathsf{napm}(y)]$$

(Proof: OTTER can derive **T.2** from **A.12** and **A.13**.)

In case of Proposition 4 we must take care of the counterexamples in Table 11 (after adding Axioms 12 and 13, Proposition 4 is still not derivable, and the same counterexamples remain). All counterexamples in Table 11 are of the form $\mathsf{cons}(1) > \mathsf{cons}(0)$ and $\mathsf{soli}(0) = \mathsf{soli}(1)$,

To restore the argumentation for Proposition 4 we only need to add an axiom stating that *the greater the consensus, the greater the solidarity* (the converse of Axiom 8):

A.14 $\forall x, y\ [\mathsf{cons}(x) > \mathsf{cons}(y)\ \rightarrow\ \mathsf{soli}(x) > \mathsf{soli}(y)]$

This takes care of all counterexamples, because a new proof attempt of Proposition 4 succeeds.

Theorem 4 *The greater the consensus, the smaller the number of rejections per deviant*:

$$\forall x, y\ [\mathsf{cons}(x) > \mathsf{cons}(y)\ \rightarrow\ \mathsf{nrpd}(y) > \mathsf{nrpd}(x)]$$

(Proof: OTTER can derive **T.4** from **A.14** and **A.10**.)

One way to view the revisions above, is to regard this as adding three extra axioms, **A.12–14**. But there is another way: the original Propositions 7, 8, and 9 can be combined with their converses, axioms **A.12**, **A.14**, and **A.13** respectively, into a revised version of these propositions presented in Table 12. Viewed in this way, we have revised our formalization of Propositions 7–9, by interpreting the natural language statements like '*the greater the solidarity, the greater the consensus*' as a logical biconditional. That is, as 'the solidarity is greater if and only if the consensus is greater.' This interpretation seems justifiable, considering the inherent ambiguity of the ordinary language statements.

Table 12: Revised formalization of Propositions 7–9.

A.7*	$\forall x, y\ [\mathsf{dlab}(x) > \mathsf{dlab}(y)\ \leftrightarrow\ \mathsf{soli}(x) > \mathsf{soli}(y)]$
A.8*	$\forall x, y\ [\mathsf{soli}(x) > \mathsf{soli}(y)\ \leftrightarrow\ \mathsf{cons}(x) > \mathsf{cons}(y)]$
A.9*	$\forall x, y\ [\mathsf{napm}(x) > \mathsf{napm}(y)\ \leftrightarrow\ \mathsf{dlab}(x) > \mathsf{dlab}(y)]$

We have now changed the axioms of the theory, so we must again pose the question: Is the formal theory (still) consistent? Our earlier model in Table 5 is no longer a model of the theory, because the stronger version of Proposition 7 (axiom **A.7***) does not hold in it. Fortunately, there are still models of our revised versions of Zetterberg's Propositions 7–10, for example the model shown in Table 13.[6]

[5]Note that the proof of **T.2$^-$** requires the background assumption **MP.1**. If we had not discovered the relevance of this assumption earlier, we could have discovered it now by investigating counterexamples to **T.2$^-$**.

[6]On domain size 2, MACE generates 66 models of the revised axioms.

Table 13: A model of Axioms **A.7***, **A.8***, **A.9***, and **A.10**.

	napm	soli	cons	dlab	nrpd
0	1	1	1	1	0
1	0	0	0	0	1

>	0	1
0	F	F
1	T	F

We can easily verify that the revised axioms hold in the model of Table 13: Axiom **A.7*** holds because the group with greater division of labor (0) has also a greater solidarity; Axiom **A.8*** holds because the group with greater solidarity (0) also has greater consensus; Axiom **A.9*** holds because the group with greater number of associates per member has a greater division of labor; and finally, Axiom **A.10** holds because the group with greater solidarity (0) has a smaller number of rejections per deviant. This revised version of Zetterberg's theory has a model, therefore it is consistent.

Now all (intended) theorems are provable: the proofs of Proposition 1, 3, 5, and 6 (and 11) are still valid because we have only added axioms (or strengthened them); Proposition 2 is derivable from 7 and 8 (both in their revised form); Proposition 4 is derivable from 8 (in its revised form) and the original Proposition 10.

Moreover, there are two more theorems derivable.

Theorem 15 *the greater the consensus, the greater the number of associates per member*:

T.15 $\forall x, y\, [\mathsf{cons}(x) > \mathsf{cons}(y) \;\rightarrow\; \mathsf{dlab}(x) > \mathsf{dlab}(y)]$

(Proof: OTTER can derive **T.15** from **A.14** and **A.12**.)

Theorem 16 *the greater the consensus, the greater the number of associates per member*:

T.16 $\forall x, y\, [\mathsf{cons}(x) > \mathsf{cons}(y) \;\rightarrow\; \mathsf{napm}(x) > \mathsf{napm}(y)]$

(Proof: OTTER can derive **T.16** from **A.14**, **A.12**, and **A.13**.)

Similar to the axioms, we can combine the Theorems 1, 2, and 3, with (new) Theorems 15, 11, and 16 respectively, as Theorems **T.1***, **T.2***, and **T.3*** in Table 14.

Table 14: A revised formalization of Theorems 1–3.

T.1*	$\forall x, y\, [\mathsf{dlab}(x) > \mathsf{dlab}(y) \;\leftrightarrow\; \mathsf{cons}(x) > \mathsf{cons}(y)]$
T.2*	$\forall x, y\, [\mathsf{soli}(x) > \mathsf{soli}(y) \;\leftrightarrow\; \mathsf{napm}(x) > \mathsf{napm}(y)]$
T.3*	$\forall x, y\, [\mathsf{napm}(x) > \mathsf{napm}(y) \;\leftrightarrow\; \mathsf{cons}(x) > \mathsf{cons}(y)]$

We have now re-formalized three of the four axioms as biconditionals, i.e., **A.7***, **A.8***, and **A.9***, which allows us to derive all intended theorems, **T.1–6**. Moreover, we can derive three of these theorems as stronger biconditionals, i.e., **T.1***, **T.2***, and **T.3***.

Adding axioms allows us to derive all propositions, including the missing Propositions 2 and 4. Moreover, the stronger formulations of Axioms 7–9, i.e., **A.7***, **A.8***, and **A.9***, also restore the intended inference patterns. For example, unlike in our initial formalization, Proposition 3 (**T.3***) can now be inferred from Proposition 2 (**T.2***) and Proposition 8 (**A.8***).

3.2.5 Proposition 10

We have now formalized three of the four axioms as biconditionals (**A.7***, **A.8***, and **A.9***), and left the remaining axiom in its original version (**A.10**). Although strictly speaking not necessary for proving the theorems, one could argue that it is more natural to formalize the fourth axiom, Proposition 10, in a similar way as the other axioms. Let us explore this option.

Assume Proposition 10 is reformalized as **A.10*** in Table 15: *the solidarity is greater if and only if the number of rejections per deviant is smaller*.

Table 15: Revised formalization of Propositions 10.

A.10*	$\forall x, y\, [\mathsf{soli}(x) > \mathsf{soli}(y) \;\leftrightarrow\; \mathsf{nrpd}(y) > \mathsf{nrpd}(x)]$

The stronger version of Proposition 10 allows for the derivations of three more theorems derivable.

Theorem 17 *the greater the number of rejections per deviant, the smaller the consensus*:

T.17 $\forall x, y\, [\mathsf{nprd}(x) > \mathsf{nprd}(y) \;\rightarrow\; \mathsf{cons}(y) > \mathsf{cons}(x)]$

(Proof: OTTER can derive **T.17** from **A.10*** and **A.8**.)

Theorem 18 *the greater the number of rejections per deviant, the smaller the division of labor*:

T.18 $\forall x, y\, [\mathsf{nprd}(x) > \mathsf{nprd}(y) \;\rightarrow\; \mathsf{dlab}(y) > \mathsf{dlab}(x)]$

(Proof: OTTER can derive **T.18** from **A.10*** and **A.12**.)

Theorem 19 *the greater the number of rejections per deviant, the smaller the number of associates per member*:

T.19 $\forall x, y\, [\mathsf{nprd}(x) > \mathsf{nprd}(y) \;\rightarrow\; \mathsf{napm}(y) > \mathsf{napm}(x)]$

(Proof: OTTER can derive **T.19** from **A.10***, **A.12**, and **A.13**.)

Again, we can combine the (new) Theorems 17, 18, and 19 with Theorems 4, 5, and 6 respectively as Theorems 4*, 5*, and 6* in Table 16:

Table 16: A revised formalization of Theorems 4–6.

T.4*	$\forall x, y\, [\mathsf{cons}(x) > \mathsf{cons}(y) \;\leftrightarrow\; \mathsf{nrpd}(y) > \mathsf{nrpd}(x)]$
T.5*	$\forall x, y\, [\mathsf{dlab}(x) > \mathsf{dlab}(y) \;\leftrightarrow\; \mathsf{nrpd}(y) > \mathsf{nrpd}(x)]$
T.6*	$\forall x, y\, [\mathsf{napm}(x) > \mathsf{napm}(y) \leftrightarrow \mathsf{nrpd}(y) > \mathsf{nrpd}(x)]$

We now have formalized all 10 propositions in the same way (both axioms **A.7*****–10*** and theorems **T.1*****–6*** as biconditionals). Moreover, the suggested deductions are sound: **T.1*** is derivable from **A.7*** and **A.8***; **T.2*** is derivable from **A.7*** and **A.9***; **T.3*** is derivable from **T.2***

The model in Table 13 is a prototypical model of the theory. There are two "ideal types" of groups (Zetterberg, 1955, p.539):

> *Mechanical groups* marked by: 1) low division of labor; 2) low solidarity; 3) small membership; and 4) strong rejection of deviates from group norms.
> *Organic groups* marked by: 1) high division of labor; 2) high solidarity; 3) large membership; and 4) little rejection of deviates.

Group 0 in the model is an organic group, and group 1 is a mechanical group.

and **A.8***; **T.4*** is derivable from **A.8*** and **A.10***; **T.5*** is derivable from **A.7*** and **A.10***; and **T.6*** is derivable from **A.9*** and **T.2***.

Is the (formal) theory still consistent? The theory is consistent if it has a model. The model in Table 13 is also a model of Axioms **A.7***, **A.8***, **A.9***, and **A.10***.[7]

3.3 Recapitulating

We have now presented four formalizations of Zetterberg's natural language theory:

1. Our initial formalization with axioms **A.7**, **A.8**, **A.9**, **A.10**, and theorems **T.1**, **T.3**, **T.5**, **T.6**, (and **T.11**).
2. A version with the same axioms **A.7**, **A.8**, **A.9**, **A.10**, background assumption **MP.1**, and more theorems singled out **T.1**, **T.2**$^-$, **T.3**, **T.5**, **T.6**, (and **T.11**).
3. A version with the revised axioms **A.7***, **A.8***, **A.9***, **A.10**, and theorems **T.1***, **T.2***, **T.3***, **T.4**, **T.5**, **T.6**.
4. A last version with all axioms **A.7***, **A.8***, **A.9***, **A.10***, and all theorems **T.1***, **T.2***, **T.3***, **T.4***, **T.5***, **T.6*** as biconditionals.

Versions 1 and 2 use the same axioms, therefore they characterize the same theory. If we would choose between these first two versions, then the exposition in version 2 is closer to the natural language exposition of the theory because it presents a version of theorem 2.

Version 3 allows us to derive all propositions, including Proposition 4, and is therefore a more natural representation of the natural language theory. The price for deriving all propositions is a set of stronger axioms, which narrows down the theory's domain of application. Version 3 allows use to derive the propositions using the suggested inferences.

In version 3 of the theory, some propositions (**T.1***, **T.2***, **T.3***, **A.7***, **A.8***, and **A.9***) are represented using biconditionals, whereas other propositions (**T.4**, **T.5**, **T.6**, and **A.10**) are represented as normal conditionals. The difference in the translation of propositions is admittedly *ad hoc*: we have reinterpreted those axioms that were necessary for deriving at least a conditional version of the theorems.

Version 4 reinterprets all axioms as biconditionals, and as a result allows for also deriving biconditional statements. Version 4 gives a natural reconstruction of Zetterberg's theory in first-order logic. It gives a uniform, formal interpretation of all propositions (both axioms and theorems), it is consistent, the derivations of theorems are sound, and all theorems are satisfiable and falsifiable.[8]

Our goal is to formalize Zetterberg's theory, and, arguably, version 4 is our best candidate: it seems the closest to the natural language version of the theory, and it satisfies all the logical criteria we formulated.

The explanatory power of version 4 comes at a price: we had to reformalize the axioms as biconditionals. These versions of the axioms are strong. Consequently, they restrict the domain of the theory. We can investigate the domain of the theory by looking at its models. On domain size two, version 4 of the theory has models isomorphic to the model presented in Table 13, and there are models in which all variable are equal for both groups.[9] There are only two different models up to isomorphism. The axioms are so strong that all five variables become virtually identical, trivializing the theory. Although version 4 of the theory is a more natural translation of the natural language wording, its strong axioms seem unrealistic. To a lesser extent, the same holds for version 3 in which four of the five variables are virtually identical. Although, version 2 of the theory does not derive a version of proposition 4, it is perhaps the most promising candidate.

4 Discussion and Conclusions

The axiomatization of scientific theories in formal logic dates back, at least, to the logical positivists (Ayer, 1959). A formal theory is defined as the deductive closure of its set of axioms and theoretical explanations and predictions correspond to deductions from the set of premises (Popper, 1959). Formal logic provides several criteria for evaluating the theory, such as the consistency of the theory and soundness of deductions—criteria traditionally associated with the context of justification. We argued that these criteria can also play an important role in the revision of a theory—an activity traditionally associated with the context of discovery. The tests we suggested do not only prove the criteria, but also give a particular derivation or model that explains why a certain criterion holds or fails to hold. Examining these proofs or models provides crucial information for revising the formal theory. Moreover, this revision may have an impact on the original theory. This can be of great importance since even a minor modification of the original theory may avoid the costs involved in the empirical testing of incorrect or irrelevant hypothesis. The criteria facilitate a piecemeal revision of the theory, resulting in a cyclic process of theory development.[10]

The process of revision is essentially interactive. We attempt to use computation support for those tasks for which computers are better equipped. For example, we

[7] On domain size 2, MACE generates 34 models of the revised axioms.

[8] For proving the satisfiability of a theorem, we have to find a model (ignoring the axioms) where it holds. All theorems are satisfied in the model in Table 13. For proving falsifiability of a theorem, we have to construct a model (ignoring the axioms) in which the theorem is does not hold. For example, the models in Table 10 still prove the falsifiability of proposition 2 (note that these models do no longer belong to the revised theory).

[9] The model presented in Table 13 is one of the 34 models on domain size 2 that are produced by the automated model generator MACE. No less than 32 of these models make all axioms (and theorems) vacuously true! In 32 models all variables are equal for both groups (both 0 or both 1 for the five variables yield $2^5 = 32$ models). In the model in Table 13 all five variables are unequal, and the isomorphic copy with 0 and 1 interchanged is the last remaining model.

[10] Much like the tetrahedron examples of (Lakatos, 1976) to which our case study in §3 shows some remarkable resemblance.

use automated reasoning tools for finding proofs or models. Notice that humans theorizers have often difficulty in finding counterexamples that are non-intended models. Theorizers tend to ignore these models since they conflict with their common-sense or with their understanding of the substantive domain. Fortunately, an automated model generator does not have these difficulties. On the other hand, a human theorizer can use this knowledge to distinguish between non-intended models and genuine counterexamples. This decision is crucial because it determines whether we need to revise the premises (in case of a non-intended model) or whether we need to revise the conjecture (in case of a genuine counterexample). This decision is difficult to make for automated systems because it would require a full axiomatization of all relevant common, background knowledge.

The product of an axiomatization attempt, a first order logic rendition of a theory, is a deductive theory. Although we advocate deductive theories, we do not want to deemphasize other modes of reasoning. Quite the contrary. Consider, for example, the step to revise the theory to account for a counterexample. Such an attempt to revise the theory is *abductive*. In fact, this step is using an extended form of abduction, since we may either decide to change the premises to explain the claim (traditional abduction, see for example Aliseda-LLera, 1997), or decide to change the claim such that it can be explained by the original premises.[11] Although the product of an axiomatization is a deductive theory, the process of axiomatizing a theory is essentially non-deductive.

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[11] Note that only in the first case we really modify the formal theory—being all logical consequences of the premise set. In the second case only the exposition of the theory changes.