Situation Calculus as Hybrid Logic: First Steps*

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Abstract. The situation calculus, originally conceived by John McCarthy, is one of the main representation languages in artificial intelligence. The original papers introducing the situation calculus also highlight the connection between the fields of artificial intelligence and philosophical logic (especially modal logics of belief, knowledge, and tense). Modal logic changed enormously since the 60s. This paper sets out to revive the connection between situation calculus and modal logic. In particular, we will show that quantified hybrid logic, *QHL*, is able to express situation calculus formulas often more natural and concise than the original formulations. The main contribution of this paper is a new quantified hybrid logic with temporal operators and action modalities, tailor-made for expressing the fluents of situation calculus.

1 Introduction

The seminal paper that McCarthy and Hayes published in 1969, *Some Philosophical Problems from the Standpoint of Artificial Intelligence*, marks a watershed in artificial intelligence. It is the key reference for one of its main representation languages—the situation calculus. We will focus here on the original version of situation calculus ([13, 14]; sometimes called the "snapshots" version, to distinguish it from other variants). The most important construct of situation calculus is—no surprise—situations. As [13] has it:

> One of the basic entities in our theory is the *situation.* Intuitively, a situation is the complete state of affairs at some instant of time. ... Since a situation is defined as a complete state of affairs, we can never describe a situation fully; and we therefore provide no notation for doing so in our theory. Instead, we state facts about situations in the language of an extended predicate calculus. Examples of such facts are 1. raining(s) meaning that it is raining in situation s.

The situations are fully informed instances of the world of which we have limited knowledge, but still occur in the object language—this is what modal logicians now call a hybrid language. Precisely the same intuition is present in the writings of Arthur Prior, the founder of temporal logic [17]. McCarthy and Hayes

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[14] praise Prior's work. They include his temporal operators into the situation calculus and they note the similarity of their use of situation variables to Prior's time-instants. But that is it. Apart from this promising beginning, the languages of situation calculus and the modal languages based on Kripke's and Prior's work have always stayed far removed from each other.

We think this is at least partly due to historical reasons. First of all, Prior's writing is notoriously difficult. Secondly, in the late 60's first order modal logic was a hot topic but the debate centered around all its philosophical problems. At that time hardly anyone saw it as a useful language for doing knowledge representation, with McCarthy and Hayes as notable exceptions. In fact, Prior is an exception too; he saw that modal logic could be used for a general (dynamic) theory of information. Another important reason was the inadequate expressive power of the available modal languages for the purposes McCarthy and Hayes had in mind. Since the late 60's, this situation has changed considerably. First and foremost, we know now that actions can be naturally represented in dynamic logic, a branch of modal logic.[1] Secondly, nowadays modal logic has become a respectable member in the field of knowledge representation, be it under the name of description logic.[2] Finally, the 90's saw the emergence of a branch of modal logic called hybrid logic which took up, or sometimes reinvented, many of Prior's ideas. E.g., seemingly unaware of Prior, Passy and Tinchev [15] argue for the introduction of names for states in dynamic logic. Hybrid logic adds to modal logic explicit reference to states, a mechanism to bind variables to states (the modal–logical term for situation), and a holds operator $@_i\phi$, allowing one to express that a formula ϕ holds *at* a state named i.

The purpose of this paper is to introduce hybrid logic to the artificial intelligence community. We will do this by showing that hybrid logic is very well suited to express what is normally formulated in the situation calculus. We have chosen for a comparison with the very first situation calculus language, from [14]. Our prime reason for choosing [14], apart from the fact that it started the field, is that one can feel their struggle with the first order language they are using. They have to introduce λ–abstraction, and all the time they introduce abbreviations to make their formulas look intuitive. These abbreviations foreshadowed a number of later technical developments in modal logic (e.g., van Benthem's celebrated standard translation into first order logic). In fact, we see McCarthy and Hayes as forerunners of the use of modal logic as a knowledge representation language and would not be surprized if they had used hybridized first order modal logic to state the situation calculus if only the right ingredients had been available when they wrote their article.

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[1] Dynamic logic originates with V. Pratt [16]. The recent monograph [10] contains many applications of dynamic logic to computer science. The rendering of a version of the situation calculus in GOLOG by Levesque, Pirri and Reiter [12] is also based on dynamic logic.

[2] Description logic [5, 8] evolved out of Brachman and Schmolze's knowledge representation language KL–ONE [6]. There are now a number of very fast DL provers for very expressive (exptime complete) languages, e.g., DLP and Racer, cf., the DL web page `http://dl.kr.org/`.

The rest of this paper is structured as follows. We start with with a brief introduction to hybrid logic. In the main part of the paper we show how to express typical situation calculus statements in hybrid logic. Here we gently introduce the notions of hybrid logic and show their use in examples. Rigorous definitions of its syntax and semantics are provided in the appendix. We end with a discussion of the presented work.

2 Hybrid logic

The rapidly growing field of hybrid logic, although rooted in the philosophical logic of Prior, is now being recognized as a tool in the field of knowledge representation. Hybrid logic has close connections with the field of description logic (cf., the page `http://dl.kr.org/` or [8]). At present, several description logic theorem provers are being adjusted to handle the full nominals of hybrid logic. These provers handle propositional hybrid fragments with an exponential time worst case complexity with surprising efficiency. The proof and model theory of propositional hybrid logic is by now understood very well [3, 2]. Recent unpublished work on first order hybrid logic indicates it has enormous advantages over first order modal logic. For instance, a complete analytic tableau system exists which also yields interpolants. One of the strong indications that something is missing in the usual formulation of first order modal logic is its failure of the interpolation property [9]. The computational and applied logic group at the University of Amsterdam is currently implementing a resolution–based theorem prover for hybrid logic. Carlos Areces maintains a web page devoted to hybrid logic at `http://www.hylo.net`. There have been a number of hybrid logic (HyLo) workshops. The next will be held as a LICS–affiliated workshop during the summer of 2002.

3 Situation calculus as hybrid logic, First steps

In this section we argue that hybrid logic is an excellently suited formalism to speak about situations and fluents. We do this by reviewing the key examples in [14] and reformulate them in hybrid logic. The hybrid language will be introduced informally and step by step. A rigorous formal definition of the resulting quantified hybrid logic can be found in the Appendix.

McCarthy and Hayes seem very much willing to suppress the situation argument in their formulas, just as in first order modal logic. This shows in all example formulas in section 2 of [14]. They find it unnatural (and going against natural language practice) to add an extra argument to each predicate symbol for the situation. For example "John loves Mary" has to be expressed as $love(j, m, s)$ where s refers to a situation. For this reason they introduce "abbreviations" in which this extra argument is suppressed. (We write this between quotes as the syntactical status of these formulas is not always clear.) Still they cannot do this in all cases because they sometimes need to refer to situations explicitly. They note the similarity with Prior's nominals:

> The use of situation variables is analogous to the use of time-instants in the calculi of world-states which Prior [17] calls *U-T* calculi. [14, p.480]

We will now show that the modern treatment of Prior's ideas which has become known under the name of *hybrid logic* provides exactly the linguistic elements that McCarthy and Hayes seemed to be searching for.

The two most important semantic constructs in the situation calculus are the *situation* and the *fluent*. A situation is the complete state of the universe at an instant of time. A fluent is a function whose domain is the set of situations. *Propositional fluents* are fluents whose range is the set of truth values $\{true,\ false\}$. *Situational fluents* are those whose range is the set of situations itself.

We start with considering propositional fluents. The key idea of situation calculus is that the meaning of every expression is a fluent. If we equate situations with the possible worlds from Kripke semantics, following the suggestion in [14, p.495], then sentences in quantified modal logic express propositional fluents. For example, the meaning of the sentence "John walks" is traditionally given as the set of possible worlds in which the sentence "John walks" is true. This set of course uniquely determines a propositional fluent.

Key idea of modal logic: Every first order modal logical sentence expresses a propositional fluent. It does so without referring explicitly to situations. In fact in traditional modal logic one can not refer to the situations (more traditionally called "worlds") in the models. Also in quantified hybrid logic (*QHL*) every sentence expresses a propositional fluent. But in addition one can refer to situations and indicate that a formula holds at a certain situation.

Names for situations and a holds operator. But McCarthy and Hayes need more expressive power than quantified modal logic has to offer. They want to be able to express "At situation s, 'John walks' holds".[3] This is not possible in quantified modal logic because it contains no machinery to refer to possible worlds.

This is where Prior's ideas and their modern treatment in the form of hybrid logic come into action. For the moment, add a second sort of variables, called *nominals*, to the language of first order logic. Every nominal is a formula, and nominals can be freely combined to form new formulas. In addition, whenever i is a nominal and ϕ is a formula, then also $@_i\phi$ (pronounce: at i, ϕ) is a formula.

The function of nominals is to *name* situations. The meaning of a nominal i—an atomic formula in hybrid logic—in a model will be the propositional fluent which is true only for the unique situation that is named by i in the model. $@_i\phi$ adds a holds–operator to first order logic: $@_i\phi$ states that the formula ϕ holds at the situation named i. Thus the meaning of $@_i\phi$ is the constant propositional fluent which sends every situation to *true* if ϕ holds at the situation named i, and every situation to false otherwise.

[3] The holds operator plays an important role in a number of knowledge representation formalisms, for instance in Allen's work on events and intervals [1] and in Kowalski's event calculus [11].

Let's consider the first example from [14, p.478]. McCarthy and Hayes want to "assert about a situation s that person p is in place x and that it is raining in place x." This is expressed by $at(p, x, s) \wedge raining(x, s)$. Not being satisfied with this notation they give two other possible equivalent notations:

$$[at(p, x) \wedge raining(x)](s) \tag{1}$$
$$[\lambda s'.at(p, x, s') \wedge raining(x, s')](s). \tag{2}$$

In *QHL* all these are expressible by different formulas without lambda abstraction. The fluent $\lambda s'.at(p, x, s') \wedge raining(x, s')$ is simply expressed in *QHL* by $at(p, x) \wedge raining(x)$. The formulas (1) and (2) are then expressed by $@_s(at(p, x) \wedge raining(x))$, an almost literal translation of the statement in natural language. Finally the original formulation is expressed by distributing $@_s$ over the conjunction as in $@_s\, at(p, x) \wedge @_s raining(x)$.

Theories and definitions. There is a second reason why McCarthy and Hayes want explicit reference to situations. To express laws of nature, definitions or other information which is supposed to be true in all situations, you have to universally quantify over situations. They give the example of a kind of transitivity for the predicate *in(x,y,s)* which expresses that x is in the location in situation s:

$$\forall x \forall y \forall z \forall s.(in(x, y, s) \wedge in(y, z, s) \rightarrow in(x, z, s)) \tag{3}$$
$$\forall x \forall y \forall z \forall .(in(x, y) \wedge in(y, z) \rightarrow in(x, z)). \tag{4}$$

In the second statement the situation argument is suppressed and $\forall.$ is meant to implicitly quantify over all situations. In modal terminology $\forall.$ functions as a *universal modality.* In description logic a special status is given to statements which are supposed to be true in all situations. They are placed in, what is called, the *T–Box* (for Theory Box). This is the natural place to collect definitions and other laws which hold universally. We adopt this T–Box machinery and express (3) and (4) simply by putting the *QHL* sentence (5) in the T–Box.

$$\forall x \forall y \forall z(in(x, y) \wedge in(y, z) \rightarrow in(x, z)) \tag{5}$$

Note that this is almost literally the formulation (4) which is preferred in [14], except that the unappealing empty quantifier is replaced by the T–Box.

Prior's temporal operators. In section 2 of [14], Prior's temporal operator F is introduced in the situation calculus. Here it becomes clear that the used formalism is not suited: only with explicit λ–abstraction can one make a simple causality assertion. $\mathsf{F}(\pi, s)$ means that "the situation s will be followed (after an unspecified time) by a situation that satisfies the fluent π". To describe the temporal aspect of situations, McCarthy and Hayes postulate a function *time* from the set of situations to a set of time–points. The last set comes with the usual (linear) *earlier than* ordering.

Now (6) is the formalization of the assertion that "if a person is out in the rain, he will get wet".

$$\forall x \forall p \forall s[raining(x, s) \wedge at(p, x, s) \wedge outside(p, s) \rightarrow \mathsf{F}(\lambda s'.wet(p, s'), s)]. \tag{6}$$

This is also too much for McCarthy and Hayes and they quickly suppress explicit mention of situations, yielding

$$\forall x \forall p \forall .[raining(x) \wedge at(p, x) \wedge outside(p) \rightarrow \mathsf{F}(wet(p))]. \tag{7}$$

If we delete the empty quantifier $\forall .$ in (7) and put the result in the T–Box, we get the formalization in temporal *QHL*.

In temporal *QHL*, Prior's temporal operators F and P are added to the language: whenever ϕ is a formula, also $\mathsf{F}\phi$ and $\mathsf{P}\phi$ are formulas. Their meaning is evaluated locally in a situation: $\mathsf{F}\phi$ is true in a situation s if there exists a situation s' such that $time(s) < time(s')$ and ϕ is true at s'. The meaning of $\mathsf{P}\phi$ is defined similarly but with s' *before* s. Thus $\mathsf{F}\phi$ is true in a situation s if there exists a situation *in the future* of s at which ϕ is true. $\mathsf{P}\phi$ expresses the same thing, but with respect to the *past*.

Actions. The largest change in the language comes from our treatment of actions as compared to that in [14]. (A related approach is taken by Levesque, Pirri and Reiter [12], cf. also Reiter's book [18]). We treat actions as in dynamic logic [10] and introduce a modality for every action. McCarthy and Hayes [14] deal with actions through the situational fluent $result(p, \sigma, s)$. In this, p is a person, σ an action and s a situation. The value of $result(p, \sigma, s)$ is the situation that results when p carries out σ, starting in s. If the action does not terminate $result(p, \sigma, s)$ is considered undefined.

Note that $result(p, \sigma, s)$ is a function with the set of situations as its range. Using functions one can only handle deterministic actions. Another drawback of this representation is the use of partial functions. It is unclear what truth value a formula should receive when some of its arguments are undefined. Reiter [18] has similar problems which lead to the introduction of "ghost situations." Dynamic logic offers a solution for these problems, but pays the price that explicit reference to situations is not possible in the language. As we will see, when this is needed it can be elegantly done in hybrid logic. To simplify matters, we just consider actions and let the actor be implicit. So assume there is a set ACT of primitive actions. Then whenever ϕ is a formula and $\alpha \in \mathsf{ACT}$ is an action, also $\langle\alpha\rangle\phi$ and $[\alpha]\phi$ are formulas. $\langle\alpha\rangle\phi$ is true in a situation s if there exists a situation s' which is the result of carrying out α in s and ϕ is true in s'. $[\alpha]\phi$ is defined dually, so that ϕ needs to be true in *all* situations s' which result from carrying out α in s. Thus if α is a deterministic action $@_s[\alpha]\phi$ expresses that ϕ is true in the situation $result(\alpha, s)$.

McCarthy and Hayes use *result* to express certain laws of ability of the form $@_s\phi \rightarrow @_{s'}\psi$ with $s' = result(\sigma, s)$, expressing that if ϕ holds at s, then ψ is true in the situation which is the result of carrying out σ in s. With action modalities one can make more fine–grained distinctions. $@_s\phi \rightarrow \langle\alpha\rangle\top$ expresses that α *can* be carried out in situation s if ϕ holds there. $@_s\phi \rightarrow [\alpha]\psi$ expresses that *if* α is carried out in s under the assumption of ϕ, then ψ is true in every resulting situation (though there need not exist one). Here are two more examples of properties which cannot be expressed in situation calculus (or for that matter, in dynamic logic), but can in the hybrid formalism:

- $@_s\langle\alpha\rangle\top$ expresses that it is possible to carry out action α successfully in situation s;
- $@_s[\alpha]\mathsf{P}s$ expresses that the situation which results after carrying out action α in situation s is later in time than s. In plain words this formula expresses that it takes time to perform α.

The combination of actions into strategies is immediate in this approach. Whenever ϕ is a formula and $\alpha_1, \ldots, \alpha_n \in \mathsf{ACT}$ are actions, also $\langle\alpha_1\rangle \cdots \langle\alpha_n\rangle\phi$ and $[\alpha_1] \cdots [\alpha_n]\phi$ are formulas.

Dynamically creating names for situations. For some applications we need to be able to refer to situations which result from carrying out actions. This can be done by the downarrow binder from hybrid logic. Intuitively $\downarrow x.\phi$ is true at a situation s if ϕ is true at s under the assumption that x refers to the situation s. A few examples will clarify its usefulness. $\downarrow x.\langle\mathsf{skip}\rangle x$ expresses that the result of performing skip in any situation named x is always the situation named x. The next formula expresses that drinking is a continuous action (meaning that every drinking action is a sequential composition of two (smaller) drinking actions)

$$\downarrow x.[\mathsf{drink}]\downarrow y.@_x\langle\mathsf{drink}\rangle\langle\mathsf{drink}\rangle y.$$

To see how this works, suppose the formula is true in situation s. Then the formula $[\mathsf{drink}]\downarrow y.@_x\langle\mathsf{drink}\rangle\langle\mathsf{drink}\rangle y$ is true in s assuming that x refers to s. Hence for all situations s' which result after drinking in s, $\downarrow y.@_x\langle\mathsf{drink}\rangle\langle\mathsf{drink}\rangle y$ is true in s'. Thus, assuming also that y refers to s', $@_x\langle\mathsf{drink}\rangle\langle\mathsf{drink}\rangle y$ is true in s'. But under the naming assumptions this is true precisely if two drinking actions performed after each other can lead from s to s'.

4 Discussion and conclusions

The seminal paper that McCarthy and Hayes [14] published in 1969 marks a watershed in artificial intelligence. Its importance can simply not be underestimated: apart from introducing the situation calculus as one of the main representation languages in artificial intelligence, the paper is most famous for singling out a number of fundamental problems that did set artificial intelligence's research agenda for years to come. Amongst its most important contributions are its role in the identification of the monotonicity of classical logic as a fundamental problem for intelligent robots; and perhaps it is most famous for introducing the frame problem (an area of unsurpassed activity in artificial intelligence). Both these fundamental problems resulted in important research traditions (see [7] for an overview of the field of non-monotonic reasoning, and see [19] for a survey of the frame problem). Nowadays, the ideas of [14] seem to have reached their ultimate success—they are part of the common knowledge and taken for granted by most researchers. Nevertheless, we feel that there are more than historical reasons for re-appraising [14].

A less frequently discussed contribution of the original paper is that it highlighted the connection between the fields of AI and philosophical logic (especially modal logics of belief, knowledge, and tense). This is even more extraordinary considering that the formulation in terms of Kripke semantics of these modal logics were recent developments in the 60s, and at that time part of a rather peripheral area in logic, plagued by deep philosophical problems. However, also modal logic progressed since the 60s and broadened its subject matter. As an illustration, the recent monograph [4] starts with stating that "modal languages are simple yet expressive languages for talking about relational structures". It is this view, of modal logic as a multi–purpose knowledge representation language, which holds the promise to shed new light on some of the fundamental problems of knowledge representation. Arthur Prior held this view already, now it is being fully developed in the fields of description logic [8] and hybrid logic [3].

The main contribution of this paper is a new quantified hybrid logic with temporal operators and action modalities, tailor-made for expressing the fluents of situation calculus. We have shown that in this quantified hybrid logic, situation calculus formulas can be expressed more natural and concise than the original formulations. Moreover, it comes with additional operators such as a downarrow binder that may enhance its expressive power beyond the original situation calculus. More generally speaking, the aim of this paper was to revive the connection between situation calculus and modal logic. This aim can perhaps best be viewed as an effort to bring back together two research traditions that have worked independently for many years. This may also help to highlight some of the common interests of knowledge representation and modal logic. We can only hope that this inspires further collaboration, and fruitful exchange of ideas between the two communities.

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Appendix: formal definition of quantified hybrid logic

The language of quantified hybrid logic *QHL* is obtained by adding nominals to name situations, the holds operator $@_s$, Prior's temporal operators F and P, and the action modalities $\langle\alpha\rangle$ and $[\alpha]$ to ordinary first order logic with equality. In detail, we have a set NOM of nominals, a set ACT of action statements, a set FVAR of first order variables, a set CON of first order constants, and predicates of any (including nullary) arity.

The *terms* of the language are the constants from CON plus the first order variables from FVAR. The *atomic formulas* are all symbols in NOM together with the usual first order atomic formulas generated from the predicate symbols and equality using the terms. *Complex formulas* are generated from these according to the rules

$@_n\phi$	holds operator
$\neg\phi \mid \phi\wedge\psi \mid \phi\vee\psi \mid \phi\rightarrow\psi$	booleans
$\exists x\phi \mid \forall x\phi$	quantifiers
$\mathsf{F}\phi \mid \mathsf{P}\phi$	temporal operators
$\langle\alpha\rangle\phi \mid [\alpha]\phi$	actions modalities.

Here $n \in \mathsf{NOM}$, $x \in \mathsf{FVAR}$, and $\alpha \in \mathsf{ACT}$.

These formulas are interpreted in situation calculus models. Such a model is a structure $(S, \mathit{time}, T, <, \{R_\alpha\}_{\alpha\in\mathsf{ACT}}, I_{nom}, D, I_{con}, I_s)_{s\in S}$ such that

- S is a set of situations
- time is a function from S to the set of time points T
- $(T, <)$ is a linearly ordered flow of time
- $\{R_\alpha\}_{\alpha\in\mathsf{ACT}}$ is a set of binary relations on S, one for each action $\alpha \in \mathsf{ACT}$.
- I_{nom} is a function assigning members of S to nominals;
- I_{con} is a function assigning elements of D to constants in CON;
- for each $s \in S$, (D, I_s) is an ordinary first order model

To interpret formulas with free variables we use assignments. An *assignment* is a function g from FVAR to D. With g^x_d we denote the assignment which is just like g except that $g(x) = d$. Given a model and an assignment g, the interpretation of terms t, denoted by $\bar{t}$, is defined as

$$\begin{aligned} \bar{x} &= g(x) \quad \text{for } x \text{ a variable} \\ \bar{c} &= I_{con}(c) \text{ for } c \text{ a constant .} \end{aligned}$$

Now we define the crucial satisfaction relation: when is a formula ϕ true in situation s in model $\mathfrak{M}$ under the assignment g. We abbreviate this by $\mathfrak{M}, g, s \Vdash \phi$. Note that this is just a handy way of defining exactly which fluents are expressed by which formulas. The definition follows the recursive construction of the language. First we define $s \Vdash_g \phi$ for the atomic cases,

$$\begin{aligned} s \Vdash_g R(t_1, \ldots, t_n) &\iff \langle \bar{t}_1, \ldots, \bar{t}_n \rangle \in I_s(R), \text{ for } R \text{ an } n\text{-ary predicate symbol} \\ s \Vdash_g t_i = t_j &\iff \bar{t}_i = \bar{t}_j \\ s \Vdash_g n &\iff I_{nom}(n) = s, \text{ for } n \text{ a nominal} \end{aligned}$$

for the holds operator,

$$s \Vdash_g @_n\phi \iff I_{nom}(n) \Vdash_g \phi \text{ for } n \text{ a nominal}$$

for the booleans,

$$\begin{aligned} s \Vdash_g \neg\phi &\iff \text{not } s \Vdash_g \phi \\ s \Vdash_g \phi \wedge \psi &\iff s \Vdash_g \phi \text{ and } s \Vdash_g \psi \\ s \Vdash_g \phi \vee \psi &\iff s \Vdash_g \phi \text{ or } s \Vdash_g \psi \\ s \Vdash_g \phi \rightarrow \psi &\iff s \Vdash_g \phi \text{ implies } s \Vdash_g \psi \end{aligned}$$

for the quantifiers,

$$\begin{aligned} s \Vdash_g \exists x\phi &\iff s \Vdash_{g^x_d} \phi, \text{ for some } d \in D \\ s \Vdash_g \forall x\phi &\iff s \Vdash_{g^x_d} \phi, \text{ for all } d \in D \end{aligned}$$

for the temporal operators,

$$\begin{aligned} s \Vdash_g \mathsf{F}\phi &\iff s' \Vdash_g \phi \text{ for some } s' \in S \text{ such that } \mathit{time}(s) < \mathit{time}(s') \\ s \Vdash_g \mathsf{P}\phi &\iff s' \Vdash_g \phi \text{ for some } s' \in S \text{ such that } \mathit{time}(s') < \mathit{time}(s) \end{aligned}$$

and for the action modalities,

$$\begin{aligned} s \Vdash_g \langle\alpha\rangle\phi &\iff t \Vdash_g \phi \text{ for some } t \in W \text{ such that } R_\alpha st \\ s \Vdash_g [\alpha]\phi &\iff t \Vdash_g \phi \text{ for all } t \in W \text{ such that } R_\alpha st. \end{aligned}$$

Let T be the T–Box which is a set of *QHL* sentences, and let ϕ be a *QHL* sentence. We say that T and ϕ are *satisfied* in a model $\mathfrak{M}$, if

- all sentences in T are true in all situations in $\mathfrak{M}$, and
- ϕ is true is some situation in $\mathfrak{M}$.

For most cases, the above language is strong enough. If explicit reference to situations obtained by an action is needed, the $\downarrow$ binder should be added. With this operator added, the language becomes virtually equivalent to the situation calculus. It is hard to state such a result in a precise way because the situation calculus itself does not have a precise boundary. Still, in the formulation of [14] it is a first order language. For this language, the relation to hybrid logic is established in [2] as follows: a first order formula $\phi(s)$ is equivalent to a hybrid formula if and only if the validity of $\phi(s)$ in a model is unaffected by adding or removing situations to the model *which cannot be reached through a finite number of actions from s*. The meaning of formula $\forall s\phi(s)$ can thus be captured by placing the hybrid formula equivalent to $\phi(s)$ in the T–Box.